Voluntary Disclosure and Informed Trading[*]

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Abstract

I study the impact of informed trading on voluntary corporate disclosure in the presence of two frictions: cost of disclosure and value of managerial information. In the absence of both frictions, informed trading has *no impact on disclosure* even when traders are not certain whether the manager has information. If disclosure is costly, then informed trading *reduces disclosure.* Since traders can discover favorable information about the firm, additional disclosure of the information is not necessary. If managerial information is valuable for the firm, then informed trading *increases disclosure.* Since traders can discover unfavorable information about the firm, the manager with such information has less incentives to pool with uninformed managers and discloses to show that he is informed. I also show that informed trading can have both a positive and a negative real effect on the firm value by *crowding in* or *crowding out* information production in the firm. These results hold for general information structures and are robust if traders can choose how much information to acquire.

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1 Introduction

Voluntary corporate disclosure is crucial for price discovery. For instance, in a survey on financial reporting Beyer et al. [2010] show that voluntary disclosures contain more information related to the stock price than mandatory disclosures and analysts' reports together. In particular, more than 80% of all price-relevant information disclosed by firms is provided voluntarily as earnings guidances, earnings warnings and management forecasts.

There is vast theoretical literature that studies managerial incentives to voluntarily disclose information to uninformed traders. Another well-developed strand of literature investigates how mandatory disclosures affect prices and information production in the financial market. However, there is almost no theoretical research on how production of information and informed trading in the financial market influence voluntary disclosures. This is surprising given that in a survey of chief financial officers Graham et al. [2005] show that more than 80% of managers disclose information voluntarily in order to reduce "information risk" associated with the stock.

In this paper, I study how informed trading affects the firm manager's incentives to produce and voluntary disclose information about the firm value. To this end, I model an economy with a firm manager, a continuum of asymmetrically informed risk-neutral traders and a risk-neutral market maker. There are two types of traders: the rational ones who receive signals about the firm asset's fundamental value and trade to maximize their profits and the "noise" traders with a uniformly distributed demand for the asset.[1] The market-maker observes the total demand and sets the price equal to the expected value of the firm asset.

The firm's manager may exert costly unobservable effort to attempt to acquire information about the firm. If he succeeds, he receives a separate signal about the firm value and can reveal it at a cost before the trade takes place. The manager chooses how much effort to exert and whether

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[1]The assumption of uniform distribution is made for tractability and can be somewhat relaxed. For example, exponential or shifted exponential distributions also give tractable and qualitatively similar results.

to disclose or not if he receives a signal. His objective is to maximize the expected price of the firm asset. I assume that the informativeness of the manager provides additional value for the firm.[2] All equilibria in the model are of a threshold type, such that the manager discloses if his signal is above some endogenous threshold and conceals otherwise.

The model in the paper nests the models of costly voluntary disclosure of Jovanovic [1982] and Verrecchia [1983] and the models of Dye [1985] and Jung and Kwon [1988] with uncertainty in the market regarding the information endowment of the manager.

I first study the setup when both the effort of the manager to acquire information and information production in the financial market are fixed. If disclosure is costless and managerial informativeness is irrelevant for the firm value, then informed trading in the market has *no effect* on the disclosure strategy even if the financial market is uncertain about information endowment of the manager. The manager with a threshold signal expects the price to be on average the same whether he discloses or not, independently of informed trading.

If disclosure is costly, then informed trading in the market increases the disclosure threshold and thus *reduces disclosure*. The intuition is as follows. The manager with a marginal signal is indifferent between disclosing his signal at a cost and concealing it. Informed traders can incorporate the managerial information into the price even when there is no firm disclosure. Consequently, the manager can essentially *free ride* on informed traders and save his cost.

If managerial information is valuable for the firm, then informed trading in the market decreases the disclosure threshold and thus *increases disclosure*. The intuition for this is as follows. The manager with a marginal signal conceals to pool with uninformed managers. However, informed trading can incorporate the managerial information into the price even when there is no firm disclosure. The efficiency of pooling with uninformed managers decreases, while the cost (not showing that the

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[2]Managerial information is crucial for the firm's operational decisions, performance evaluation and control (see e.g. Baiman and Demski [1980] or Arya et al. [1997]). Also, managerial informativeness signals high ability and effort of the manager, which are incremental for the firm value (Chemmanur and Paeglis [2005]).

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manager is informed and thus valuable for the firm) remains the same. Consequently, the manager discloses more information.

When both the disclosure is costly and the managerial information is valuable, then the net effect of informed trading on disclosure is dubious. If the cost of disclosure is sufficiently high, then the first effect dominates, so that informed trading reduces disclosure. If the value of managerial information is sufficiently large, then the second effect prevails, so that informed trading increases disclosure. I provide conditions under which the net effect is positive and negative.

Then, I extend the model by allowing traders and the firm manager to optimally acquire costly information. First, if informed traders in the market can optimally choose how much information to obtain, then all results remain robust. Second, I allow the manager to optimally choose how much costly unobservable effort to exert to acquire information. If the cost of disclosure is sufficiently larger than the value of managerial information, then informed trading in the market *crowds out* both information production in the firm and the firm disclosure. Intuitively, when the cost of disclosure is high, then informed trading decreases disclosure, so there are less incentives to collect information in the first place. However, if the cost of disclosure is sufficiently smaller than the value of managerial information, then informed trading in the market can also *crowd in* information production in the firm. Intuitively, when the value of managerial information is high, then informed trading increases disclosure, so there are more incentives to collect information in the first place. In the model, informed trading increases the firm value if and only if it stimulates information production in the firm. Consequently, the real effect of informed trading in the model can be both positive and negative. These results remain robust when both the information acquisition in the firm and in the market are optimal.

The rest of the paper proceeds as follows: Section 2 reviews the literature. Section 3 lays out the model. Section 4 derives the results in the case without information acquisition by traders or

the firm manager. Sections 5 and 6 extend the model by allowing traders and the firm manager to optimally acquire their information. I provide empirical predictions in Section 7. Section 8 concludes.

2 Literature Review

The paper merges two strands of literature: voluntary disclosure and mandatory disclosure to the market with informed trading. Literature on voluntary disclosure studies managerial incentives to reveal information to outside agents such as investors or competitors. The "unraveling result" of Grossman and Hart [1980] and Milgrom [1981] determines conditions under which managers optimally disclose all their private information to outsiders. Verrecchia [2001], Dye [2001] and Beyer et al. [2010] survey models of voluntary disclosure in which those conditions are relaxed. For example, Dye [1985] and Jung and Kwon [1988] relax the assumption that it is a common knowledge that managers have private information and Jovanovic [1982] and Verrecchia [1983] relax the assumption of costless disclosure. They show that the manager optimally reveals signals above a certain threshold and conceals otherwise. Jorgensen and Kirschenheiter [2003] and Heinle and Smith [2015] study voluntary disclosure of firm risk. Bagnoli and Watts [2007] explore voluntary disclosure in the case when private information of the managers compliments the mandatory disclosure of the firm. Beyer and Guttman [2012] show that voluntary disclosures are jointly determined with investment strategies and Cheynel [2013] links voluntary disclosure to the systematic risk of firm assets. Einhorn and Ziv [2008] and Guttman et al. [2014] study voluntary disclosure in a dynamic setup.

The second strand of literature studies how mandatory disclosure influences informed trading in the stock market. Diamond [1985] and Diamond and Verrecchia [1991] find that disclosure can alleviate information asymmetry among traders. Kim and Verrecchia [1991], Kim and Verrecchia

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[1994], McNichols and Trueman [1994] and Demski and Feltham [1994] study how exogenous public disclosures affect information asymmetry through endogenous information acquisition by traders. In particular, they discuss how information in the market can crowd in firm disclosure. In a model with multiple firms, Easley and O'hara [2004] show that stocks with greater private information are traded at a discount and Lambert et al. [2007] show that this information effect is diversified away when the number of traders becomes large. Gao [2010] studies how increased quality of public disclosure in a production economy can make investors worse off even though the cost of capital decreases. Kumar et al. [2016] study how the presence of an informed trader changes voluntary disclosure and capital investment in an initial public offering of the firm.

Bushman and Indjejikian [1995] build a model of voluntary disclosure when a corporate insider can trade in the market. Gao and Liang [2013] and Tang [2014] build models of disclosure with endogenous quality in a financial market with information asymmetry and endogenous information acquisition. However, these three papers study voluntary disclosure with commitment. In contrast, I study the voluntary disclosure of the manager who cannot commit to a disclosure strategy ex-ante.

Dye [1998] and Bertomeu et al. [2011] study disclosure to the market, in which the traders can obtain information about the information endowment of the firm manager. In contrast, I study how disclosure is affected by traders who acquire information about the fundamental value of the firm.

To the best of my knowledge, only Ronen and Yaari [2002] study voluntary disclosure in a model with asymmetrically informed traders who can acquire private information about the fundamental. In their model, the voluntary disclosure of a firm can reflect the manager's effort and thus the manager has incentives to bias his report. They focus on optimal firm manager's compensation. To this end, they constrain the set of signals of the manager and informed traders to a binary set. In their model, each equilibrium either has full disclosure or full non-disclosure of the firm

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manager's signal. In contrast, the focus of this paper is to study truthful voluntary disclosure and information production in the firm in a standard setting of Jovanovic [1982], Verrecchia [1983], Dye [1985], and Jung and Kwon [1988] . The only assumption imposed on the distributions of signals of traders and the firm manager is the monotone likelihood ratio property. Also, both disclosure and non-disclosure are equilibrium outcomes.

3   Model

There is one firm in the economy with a manager and a risky asset of a fundamental value $\theta \in \{0, 1\}$. The firm asset is traded in the financial market which consists of a mass 1 of rational risk-neutral traders, noise traders and a risk-neutral market maker.

Before the trade takes place, the firm manager receives a signal s_F about the firm fundamental value θ with some probability g. The information endowment of the manager $\mathcal{F}_{firm}$ is

$$
\mathcal{F}_{firm} = \begin{cases} s_F, & \text{with probability } g, \\ \\ NI, & \text{with probability } 1 - g, \end{cases}
$$

where NI denotes a "not informed" firm manager. In the following, I also denote by I an informed firm manager. In the baseline model solved in Section 4, the probability of receiving a signal g is an exogenous parameter. I relax this assumption in Section 6 by allowing the manager to optimally exert effort to increase the probability of becoming informed.

Information about the firm fundamental value θ allows managers to make more efficient production and investment decisions. To model this in a parsimonious way, I assume that an informed firm manager brings additional value $\gamma \in [0, 1]$ to the firm.[3] The total fundamental value of the

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[3] The value of managerial information γ can of course depend on the fundamental value of the firm θ. For example, the value of informed management in the crisis $\gamma(\theta = 0)$ may be larger than in the boom $\gamma(\theta = 1)$ or vice versa. However, the results in the paper are robust to such refinement and I do not present them.

firm is

$$
V = \begin{cases} \theta + \gamma\,, & \mathcal{F}_{firm} = I\,, \\[2ex] \theta\,, & \mathcal{F}_{firm} = NI\,. \end{cases}
$$

If the manager receives the signal, he can disclose it to the whole market (all traders and the market maker) at a cost c_D. Information provided to the market after disclosure or non-disclosure is

$$
\mathcal{F}_{public} = \begin{cases} s_F\,, & \text{disclosure}\,, \\[2ex] ND\,, & \text{no disclosure}\,, \end{cases}
$$

where ND denotes "no disclosure". The informed manager decides to disclose if the expected price of the firm asset after disclosure net of the cost of disclosure is larger than the expected price of the firm asset after non-disclosure:

$$
\mathbb{E}[P|s_F,\, \mathcal{F}_{public} = s_F] - c_D \;\geq\; \mathbb{E}[P|s_F,\, \mathcal{F}_{public} = ND]\,.
$$

The manager's signal s_F is distributed on $\mathbb{R}$ with a density $f(s_F|\theta)$ conditional on the firm fundamental θ (the corresponding cumulative distribution function is $F(s_F|\theta)$). I assume that the signal density satisfies the monotone likelihood ratio property:

$$
\left.\frac{f(s_F|\theta = 1)}{f(s_F|\theta = 0)}\right|'_{s_F} > 0.
$$

The property guarantees that larger signals are more indicative of a high firm value $\theta = 1$.

The setup nests both the models of Dye [1985] and Jung and Kwon [1988] with an uncertainty of the market regarding the information endowment of the manager ($g < 1$) and Jovanovic [1982] and Verrecchia [1983] with costly disclosure of proprietary information ($c_D > 0$).

After the disclosure or non-disclosure of the firm, each rational trader i observes an additional

signal s_i distributed on $\mathbb{R}$ with a density $\psi_i(s_i|\theta)$ conditional on the firm fundamental θ (the corresponding cumulative distribution function is $\Psi_i(s_F|\theta)$). I assume that signal densities of all traders satisfy the monotone likelihood ratio property:

$$\left.\frac{\psi_i(s_i|\theta=1)}{\psi_i(s_i|\theta=0)}\right|'_{s_i} > 0, \qquad \forall i \in [0, 1].$$

In the baseline model solved in Section 4, the distribution functions $\psi_i(s_i|\theta)$ are all exogenous. I relax this assumption in Section 5 by allowing managers to optimally choose distribution functions of their signals.

After observing a signal s_i, each trader i submits a market order to buy x_i units of the asset to the market maker. Traders are subject to portfolio constraints $x_i \in [-1, 1]$, so that they cannot buy or short-sell more than 1 unit of the firm asset.[4] There are also noise traders with a total demand of the firm asset x_N which is unobservable and uniformly distributed, $x_N \in [-K, K]$.[5] If $K < 1$ then the noise in the market is too small, and the price is always fully revealing independently of firm disclosure. In what follows, I assume that $K > 1$, so that the price is not always fully revealing.

A risk-neutral market maker observes the total demand x from rational and noise traders,

$$x = \int_0^1 x_i \, di + x_N,$$

and sets the price of the firm asset P equal to the expected total value V of the firm conditional on x and firm disclosure or non-disclosure

$$P = \mathbb{E}[V|x, \mathcal{F}_{public}].$$

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[4]The assumption is a particular form of limits to arbitrage and is common in the finance literature.

[5]I make the assumption of uniform noise demand for tractability. Tractable and qualitatively similar results can also be obtained if the distribution of noise demand x_N or noise supply $-x_N$ are exponential or shifted exponential.

At the last stage of the game, the total value V is realized.

Figure 1 summarizes the timeline of the model. Section 4 solves the model in the case when the information acquisition in the financial market is fixed (distribution functions $\psi_i(s_i|\theta)$ are exogenously given) and information production in the firm is fixed (probability of receiving a signal g is an exogenous parameter). Section 5 relaxes the first assumption (introduces the dashed blue block in Figure 1). Section 6 relaxes the second assumption (introduces the dotted red block in Figure 1).
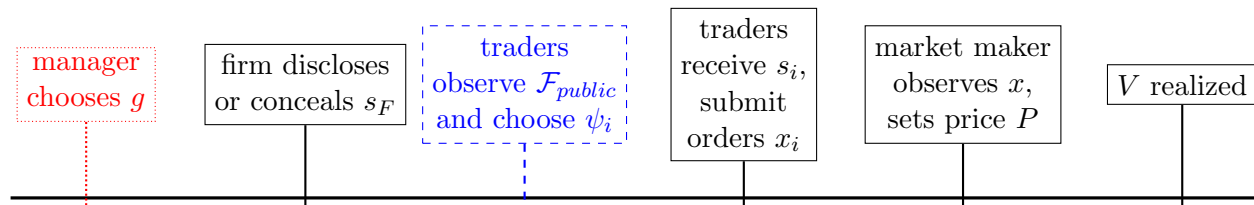

Figure 1: Timeline

4 Equilibrium

For now, let us assume that the distribution functions $\psi_i(s_i|\theta)$ of informed traders' signals are exogenously given and equal,

$$\psi_i(s_i|\theta) = \psi(s_i|\theta),$$

so that all traders are identical ex-ante before receiving signals s_i. Given that traders are risk-neutral, I conjecture (and later prove) that the optimal trading strategy of each trader i is of a "bang-bang" form.

Conjecture 1 (Optimal Portfolio) *There exists a threshold S such that the trading strategy of*

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each trader i is

$$
x_i \;=\; \begin{cases} -1\,, & s_i < S\,, \\[2mm] 1\,, & s_i \geq S\,. \end{cases}
$$

The threshold S satisfies

$$
\frac{\psi(S|\theta = 0)}{\psi(S|\theta = 1)} \;=\; 1\,.
$$

This trading strategy is intuitive: risk-neutral traders always buy as many firm assets as they can ($x_i = 1$) if they expect that the fundamental value of the firm asset is high ($\theta = 1$) with sufficiently high probability ($s_i > S$) and short-sell otherwise. At the threshold signal $s_i = S$, the conditional density functions $\psi(s_i|\theta)$ coincide and the trader is indifferent between buying and short-selling the asset.

Conditional on the firm asset's fundamental value θ, the total demand of rational traders is

$$
\begin{aligned}
\int_0^1 x_i \, di \;&=\; \int_{-\infty}^{+\infty} x_i(s_i)\psi(s_i|\theta)\, ds_i \\
&=\; \int_{-\infty}^{S} (-1)\psi(s_i|\theta)\, ds_i + \int_{S}^{+\infty} 1\psi(s_i|\theta)\, ds_i \\
&=\; 1 - 2\Psi(S|\theta)\,,
\end{aligned}
$$

and it perfectly reveals the firm asset value θ. However, the market maker observes a total demand from both rational and noise traders

$$
x \;=\; \int_0^1 x_i \, di + x_N \;=\; 1 - 2\Psi(S|\theta) + x_N\,,
$$

which is a noisy signal about the true fundamental value θ of the firm. The market maker sets the total price to the expected total value V, which consists of the expected fundamental value θ and

the expected value of managerial information:

$$P = \mathbb{E}[\theta|x, \mathcal{F}_{public}] + \gamma \operatorname{Prob}[I|x, \mathcal{F}_{public}].$$

If the firm manager discloses his signal, then the market maker infers that the manager is informed, i.e. $\operatorname{Prob}[I|x, \mathcal{F}_{public} \neq ND] = 1$. In contrast, if there is no disclosure, then the market maker updates his belief about the informativeness of the manager based on the total demand x. The following lemma defines the price that a market maker sets as a function of the observed demand x and public information $\mathcal{F}_{public}$ from the firm manager.

Lemma 4.1 (Price Function) *The price P of the firm asset is*

$$P(x, \mathcal{F}_{public}) = \begin{cases} \gamma \operatorname{Prob}(I|\theta = 0, \mathcal{F}_{public}), & x \leq \underline{x}, \\ \\ \mathbb{E}(\theta|\mathcal{F}_{public}) + \gamma \operatorname{Prob}(I|\mathcal{F}_{public}), & x \in (\underline{x}, \bar{x}), \\ \\ 1 + \gamma \operatorname{Prob}(I|\theta = 1, \mathcal{F}_{public}), & x \geq \bar{x} \end{cases}$$

where

$$\underline{x} = 1 - 2\Psi(S|\theta = 1) - K,$$

$$\bar{x} = 1 - 2\Psi(S|\theta = 0) + K.$$

If the total demand is sufficiently low $(x \leq \underline{x})$, then the market maker perfectly infers the value $\theta = 0$. Similarly, if the total demand is sufficiently high $(x \geq \bar{x})$, then he perfectly infers that the fundamental is $\theta = 1$. However, for the intermediate values of the total demand x the market maker cannot infer the value θ perfectly. Moreover, the total demand does not provide any additional information about θ (this is a property of uniform noise distribution). The market maker sets the price equal to the expected firm value θ and the expected value of managerial information given

the (non)disclosure information $\mathcal{F}_{public}$ from the firm manager.

Observe that if the manager discloses his information, then the market maker perfectly infers that the manager is informed and the price function simplifies to

$$P\left(x, \mathcal{F}_{public} = s_F\right) = \begin{cases} \gamma, & x \leq \underline{x}, \\ \mathbb{E}(\theta|s_F) + \gamma, & x \in (\underline{x}, \bar{x}), \\ 1 + \gamma, & x \geq \bar{x}. \end{cases}$$

The probability that informed trading reveals the fundamental value of the firm asset in a high or low state θ is

$$\mathrm{Prob}(x \leq \underline{x}|\theta = 0) = \mathrm{Prob}(x \geq \bar{x}|\theta = 1) = \frac{\Psi(S|\theta = 0) - \Psi(S|\theta = 1)}{K}.$$

The denominator K is half the length of the support in the noise traders' total demand $x_N \in [-K, K]$. It is thus a parameter which reflects the volatility of the noise traders' demand. Intuitively, as K increases the noise traders' demand becomes more volatile, and the price reveals the firm value θ less often. In contrast, the numerator $\Psi(S|\theta = 0) - \Psi(S|\theta = 1)$ captures the information quality of informed traders. A higher information quality thus increases the probability that the price fully reflects the true fundamental θ. I study the numerator in more details and provide its economic sense in the lemma below.

Lemma 4.2 (Information Quality) *Information quality δ of traders is*

$$\delta \equiv \Psi(S|\theta = 0) - \Psi(S|\theta = 1),$$

where S is the optimal threshold of the informed traders' strategy.

- *Information quality δ is 1 minus probabilities that the trader makes mistakes (buys when the fundamental is low and short-sells when it is high):*

$$\delta \;=\; 1 - \big[\mathrm{Prob}(x_i = 1|\theta = 0) + \mathrm{Prob}(x_i = -1|\theta = 1)\big].$$

- *Information quality $\delta \in (0,\, 1)$. If $\delta \to 0$, then the signal s_i is a pure noise. If $\delta \to 1$, then the signal s_i perfectly reveals the fundamental asset value θ.*

The information quality δ captures how often traders make a mistake when they trade on their signals. If the information quality is low δ, then traders' signals are noisy and traders make mistakes more often by buying an overpriced asset with $\theta = 0$ or short-selling an underpriced asset with $\theta = 1$.

In equilibrium, the conjecture about the informed traders' trading strategies is correct:

Lemma 4.3 (Optimal Portfolio) *Conjecture 1 is correct.*

This finishes the discussion of the price-formation and trade in the financial market. We can now proceed to finding the optimal disclosure strategy of the firm manager.

4.1 Firm Disclosure

The firm manager with a signal s_F chooses to disclose if and only if he expects the price after disclosure of s_F net of the cost of disclosure c_D to be at least as high as the price if there is no disclosure:

$$\mathbb{E}[P|s_F,\, \mathcal{F}_{public} = s_F] - c_D \;\geq\; \mathbb{E}[P|s_F,\, \mathcal{F}_{public} = ND].$$

This leads us to the following proposition.

Proposition 4.1 (Firm Disclosure) *The firm manager discloses his signal s_F if and only if*

$$\mathbb{E}(\theta|\mathcal{F}_{public} = s_F) \geq \mathbb{E}(\theta|\mathcal{F}_{public} = ND) + \frac{c_D}{1 - \frac{\delta}{K}} - \xi,$$

where

$$\xi = \gamma \left(\frac{1 - \frac{\delta}{K}\,\text{Prob}\,(I|ND, s_F)}{1 - \frac{\delta}{K}} - \text{Prob}\,(I|ND) \right) \geq 0.$$

If γ or $\frac{\delta}{K}$ is sufficiently small or g is sufficiently close to 0 or 1, then any disclosure equilibrium is of a threshold type such that there exists $T \in \mathbb{R}$ and the manager discloses if and only if

$$s_F \geq T,$$

and conceals otherwise.

Observe that the information quality of traders δ defined in Lemma 4.2 is the summary statistic of distributions of informed traders' signals which contains all the information relevant for the disclosure decision of the firm. Let us now discuss the optimal disclosure of the firm in details.

If disclosure is costless ($c_D = 0$), managerial information is useless for the firm ($\gamma = 0$), and there is no informed trading in the market ($\delta = 0$), then the model nests the setup of Dye [1985] and Jung and Kwon [1988] with uncertainty about information endowment of the manager. In particular, the manager discloses his signal s_F if and only if the market expectation of θ after disclosing s_F is higher than after concealing it:

$$\mathbb{E}(\theta|\mathcal{F}_{public} = s_F) \geq \mathbb{E}(\theta|\mathcal{F}_{public} = ND).$$

Observe that in this particular case, informed trading $\delta > 0$ has no effect on the disclosure policy of the firm. In particular, the models of Dye [1985] and Jung and Kwon [1988] are robust

to introduction of informed trading in the financial market.

Now let us discuss how informed trading affects voluntary disclosure when disclosure is costly or managerial information is valuable for the firm.

4.1.1 Costly Disclosure

If managerial information is useless for the firm ($\gamma = 0$) and there is no informed trading in the market ($\delta = 0$) but disclosure is costly ($c_D > 0$), then the model nests the setup of Jovanovic [1982] and Verrecchia [1983] with costly disclosure. In particular, the manager discloses his signal s_F if and only if the market expectation of θ after disclosing s_F net of the cost of disclosure is higher than after concealing it:

$$\mathbb{E}(\theta | \mathcal{F}_{public} = s_F) \geq \mathbb{E}(\theta | \mathcal{F}_{public} = ND) + c_D \,.$$

If managerial information is useless for the firm ($\gamma = 0$) but disclosure is costly ($c_D > 0$) and there is informed trading in the market ($\delta > 0$), then the manager discloses if and only if

$$\mathbb{E}(\theta | \mathcal{F}_{public} = s_F) \geq \mathbb{E}(\theta | \mathcal{F}_{public} = ND) + \frac{c_D}{1 - \frac{\delta}{K}} \,,$$

Interestingly, the effective cost of disclosure $\frac{c_D}{1 - \frac{\delta}{K}}$ increases in the information quality δ of traders and decreases in the noise volatility parameter K. The intuition for this is as follows. The manager with a marginal signal is indifferent between disclosing his signal at a cost and concealing it. Informed trading can incorporate the managerial information into the price even when there is no firm disclosure. If the price reflects the fundamental asset value θ better on average (if information quality δ of traders is higher or the volatility of noise K is lower), then the manager is less willing to disclose. He behaves as if c_D is higher. The manager essentially *free-rides* on informed traders

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and saves his cost c_D by not disclosing.

4.1.2 Valuable Managerial Information

When disclosure is costless ($c_D = 0$), but managerial information is valuable for the firm ($\gamma > 0$), then the firm manager discloses if and only if

$$\mathbb{E}(\theta|\mathcal{F}_{public} = s_F) \ \geq \ \mathbb{E}(\theta|\mathcal{F}_{public} = ND) - \xi \,,$$

where $\xi > 0$. Intuitively, if there is no disclosure, then the financial market does not know whether the manager is informed, and the price of the firm asset is discounted. Consequently, the manager has an additional incentive to disclose more information. The factor $\xi > 0$ captures this incentive. It increases in the information quality δ of traders and decreases in the noise volatility parameter K: if $\gamma > 0$ then

$$\frac{\partial \xi}{\partial \delta} \ > \ 0 \,,$$
$$\frac{\partial \xi}{\partial K} \ < \ 0 \,,$$

The intuition for this is as follows. The only reason for the manager to conceal a bad signal is to pool with uninformed managers. However, if the information quality δ of informed traders is larger or the volatility of noise in the market is lower, then the fundamental value of θ is revealed more often and pooling becomes less efficient. The benefit of concealing a signal decreases, while the cost (not showing that the manager is informed) remains the same. As a consequence, there are more incentives to disclose.

4.1.3 Costly Disclosure and Valuable Managerial Information

When both disclosure is costly and the managerial information is valuable for the firm, then the effect of informed trading on the disclosure strategy of the firm is dubious.

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On the one hand, if disclosure is costly, then informed trading discourages the firm manager from disclosing his information: informed traders incorporate information about the firm fundamental in the price and the manager free-rides by not disclosing and saving the cost of disclosure. On the other hand, if managerial information is valuable for the firm, then informed trading encourages the firm manager to disclose his information: the price reflects the fundamental more often and the incentive to pool with uninformed managers decreases.

When c_D is sufficiently large, then the effect of costly disclosure dominates and the manager discloses *less* information if the information quality of traders δ increases. When γ is sufficiently large, then the effect of value of managerial information dominates and the manager discloses *more* information if the information quality of traders δ increases. I summarize the findings in the following corollary.

Corollary 4.1 (Comparative Statics) *Let the cost of disclosure c_D decrease disclosure, i.e.*

$$\frac{\partial T}{\partial c_D} > 0.$$

- *If c_D is sufficiently large compared to γ,*

$$\frac{c_D}{\gamma} > \mathrm{Prob}\left(NI|ND, s_F = T\right),$$

then informed trading decreases disclosure and noise in the market K increases disclosure:

$$\frac{\partial T}{\partial \delta} > 0, \quad \frac{\partial T}{\partial K} < 0.$$

- If c_D is sufficiently small compared to γ,

$$\frac{c_D}{\gamma} \; < \; \mathrm{Prob}\left(NI|ND, s_F = T\right),$$

then informed trading increases disclosure and noise in the market K decreases disclosure:

$$\frac{\partial T}{\partial \delta} \; < \; 0\,, \quad \frac{\partial T}{\partial K} \; > \; 0\,.$$

Observe that I impose an assumption that the cost of disclosure decreases disclosure. This is a very intuitive assumption, however there exist equilibria in which this does not hold. In such equilibria, all the results of the corollary are reversed. Also, there might be multiple threshold equilibria (multiple T) depending on the properties of the firm signal distributions $f(s_F|\theta)$. However, Corollary 4.1 holds in all equilibria in which the cost of disclosure decreases disclosure.
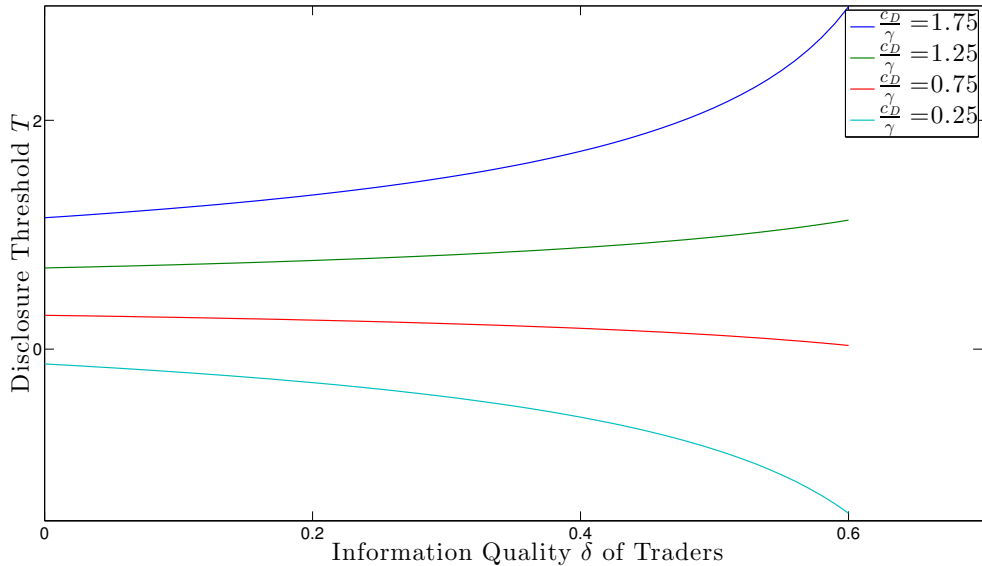

Figure 2: Threshold T's dependence on information quality δ of traders.
Parameters: $s_F\big|_\theta \sim \mathcal{N}(\theta, 1)$; $g = 0.1$; $\mathrm{Prob}(\theta = 1) = 0.5$; $K = 1$; $\gamma = 0.2$

FIgure 2 visually summarizes how the disclosure threshold T depends on the information quality

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δ of traders in the market for different values of $\frac{c_D}{\gamma}$. The net effect of informed trading on firm's voluntary disclosure is negative when the cost of disclosure is high and positive when the value of managerial information is large.

5   Optimal Information Acquisition in the Market

Now let us study the setup in which traders optimally choose how much information to obtain. Suppose that each trader i chooses the conditional density functions of her signal $\psi_i(s_i|\theta = 0)$ and $\psi_i(s_i|\theta = 1)$ after disclosure or non-disclosure of the firm (dashed blue block in Figure 1). I consider only symmetric equilibria in which all traders choose the same functions. Let us study the optimal choice $\psi(s_i|\theta = 0)$ and $\psi(s_i|\theta = 1)$ of a trader given her beliefs that all other traders choose functions $\psi^*(s_i|\theta = 0)$ and $\psi^*(s_i|\theta = 1)$.

Lemma 5.1 (Informed Trader's Profit) *Let all traders choose the same functions $\psi^*(s_i|\theta = 0)$ and $\psi^*(s_i|\theta = 1)$ of conditional distributions of their signals s_i. The expected profit of a trader who deviates to $\psi(s_i|\theta = 0)$ and $\psi(s_i|\theta = 1)$ is*

$$\mathbb{E}[\text{Profit}_i \,|\mathcal{F}_{public}] \;=\; 2\delta\left(1 - \frac{\delta^*}{K}\right)\mathbb{V}\text{ar}(\theta|\mathcal{F}_{public})M(\gamma, \mathcal{F}_{public}),$$

where δ and δ^* are the qualities of information of the trader who deviates and all other traders respectively and

$$M(\gamma, \mathcal{F}_{public}) \;=\; 1 - \gamma\left[\text{Prob}(I|\theta = 0, \mathcal{F}_{public}) - \text{Prob}(I|\theta = 1, \mathcal{F}_{public})\right] \in (0, \, 1].$$

A higher information quality δ of the signal of the trader increases her expected profit. A higher information quality of other traders δ^* or lower volatility of noise (lower K) decreases her profit

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because the price better reflects the fundamental value. Also, the profit increases in the variance of the firm fundamental θ after firm disclosure or non-disclosure $\mathbb{V}\mathrm{ar}(\theta|\mathcal{F}_{public})$, where

$$\mathbb{V}\mathrm{ar}(\theta|\mathcal{F}_{public}) \;=\; \mathrm{Prob}(\theta = 1|\mathcal{F}_{public})\,(1 - \mathrm{Prob}(\theta = 1|\mathcal{F}_{public}))\;.$$

Intuitively, a higher uncertainty in the market is beneficial for the informed trader because it increases the importance of her information, and thus the profits that she can make. If the fundamental value of the asset θ is a common knowledge before trade $(\mathrm{Prob}(\theta = 1|\mathcal{F}_{public}) \in \{0,\,1\})$, then the expected profit is 0.

If there is disclosure, then the factor

$$M(\gamma,\, s_F) \;=\; 1$$

and the value of managerial information γ is irrelevant for the expected profit of the trader. Intuitively, following disclosure of the manager, his informativeness becomes common knowledge and does not change information asymmetry between the market maker and informed traders. However, if there is no disclosure, the expected profit of the informed trader decreases in the value of managerial information γ,

$$\frac{\partial\, M(\gamma,\, ND)}{\partial\, \gamma} \;<\; 0\,.$$

The intuition for this is the following. The informed trader buys when her signal is sufficiently high and short-sells otherwise. However, if her signal is high, but there is no firm disclosure, the informed trader rationally updates the probability that the manager is informed downwards. The market maker does not observe the signal and does not update the probability. As a consequence, on average traders buy an asset which they believe to be overpriced in terms of the value of managerial information γ.

Observe that the information quality δ is the summary statistic which contains all the information about the distribution of a trader's signal relevant for her expected profit. It is natural to assume that the cost of acquiring information of quality δ is an increasing convex function

$$C_I(\delta) = \frac{c_I \delta^2}{2},$$

where $c_I > \frac{1}{2}\left(1 - \frac{1}{K}\right)$ is the cost parameter. This assumption on c_I guarantees that the equilibrium value of δ is well-defined, i.e. $\delta \in (0,1)$. After disclosure or non-disclosure, each trader chooses quality of her information δ which maximizes her expected profit net of the cost:

$$\max_{\delta \in [0,1]} \mathbb{E}[\text{Profit}_i \,|\mathcal{F}_{public}] - C_I(\delta).$$

Lemma 5.2 (Information Acquisition in the Market)

The equilibrium information quality is

$$\delta = \frac{1}{\frac{1}{K} + \frac{c_I}{2\mathbb{V}\text{ar}(\theta|\mathcal{F}_{public})M(\gamma,\mathcal{F}_{public})}}.$$

The information quality of traders δ increases in the volatility of noise traders' demand K and the uncertainty about the value of the firm's fundamental value $\mathbb{V}\text{ar}(\theta|\mathcal{F}_{public})$, and decreases in the cost of information acquisition c_I and the value of managerial information γ. The intuition is the same as for the expected profit in Lemma 5.1 discussed above.

Let us now study the optimal disclosure strategy of the firm.

5.1 Firm Disclosure

The firm manager with a signal s_F chooses to disclose if and only if he expects the price after disclosure net of the cost c_D to be at least as high as the price in the case when there is no disclosure:

$$\mathbb{E}[P|s_F, \mathcal{F}_{public} = s_F] - c_D \geq \mathbb{E}[P|s_F, \mathcal{F}_{public} = ND].$$

In contrast to the model with fixed information acquisition developed in Section 4, the manager takes into account that his disclosure changes the incentives of traders to acquire information.

Proposition 5.1 (Firm Disclosure with Optimal Information Acquisition in the Market)
The firm manager discloses his signal s_F if and only if

$$\begin{aligned}
\mathbb{E}(\theta|\mathcal{F}_{public} = s_F) \geq\ & \mathbb{E}(\theta|\mathcal{F}_{public} = ND) \\
& + \left(c_D - \gamma \operatorname{Prob}(NI|ND, s_F)\right)\left(1 + \frac{2\mathbb{V}\mathrm{ar}(\theta|ND)M(\gamma, ND)}{c_I K}\right) \\
& - \gamma\big[\operatorname{Prob}(NI|ND) - \operatorname{Prob}(NI|ND, s_F)\big].
\end{aligned}$$

If γ is sufficiently small, K or c_I is sufficiently large or g is sufficiently close to 0 or 1, then any equilibrium is of a threshold type such that there exists $T \in \mathbb{R}$ and the manager discloses if and only if

$$s_F \geq T,$$

and conceals otherwise.

As in the model with fixed information acquisition developed in Section 4, there are two channels through which informed trading affects the disclosure. On the one hand, informed trading decreases disclosure because the manager free-rides on informed traders who incorporate fundamental information in the price. On the other hand, informed trading increases disclosure because

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pooling with uninformed managers becomes less efficient.

Both effects depend on informed trading in the case of no disclosure. Consequently, only the information acquisition following no disclosure is relevant for the disclosure policy of the manager.

When the cost of disclosure c_D is sufficiently large, then the effect of costly disclosure dominates and the manager discloses *more* information if private information acquisition is more costly (c_I is larger). When γ is sufficiently large, then the effect of valuable managerial information dominates and the manager discloses *less* information if private information acquisition is more costly (c_I is larger).

There are two effects in the comparative statics with respect to volatility K of noise in the total demand. On the one hand, a higher volatility of noise makes the price less transparent, so that the information of informed traders is worse reflected in the price. On the other hand, higher K encourages informed traders to acquire costly information because their expected profits go up. Consequently, higher information quality δ makes the price more transparent. In the model, the first effect always dominates: a higher noise K always makes the price less transparent.

I summarize the findings in the following corollary.

Corollary 5.1 (Comparative Statics with Optimal Information Acquisition in the Market)
Let the cost of disclosure c_D decrease disclosure, i.e.

$$\frac{\partial T}{\partial c_D} \; > \; 0 \, .$$

- *If c_D is sufficiently large compared to γ,*

$$\frac{c_D}{\gamma} \; > \; \mathrm{Prob}\left(NI|ND, s_F = T\right),$$

  then both the cost of information acquisition in the market c_I and the noise in the market K

increase disclosure:

$$\frac{\partial T}{\partial c_I} \; < \; 0 \, , \quad \frac{\partial T}{\partial K} \; < \; 0 \, .$$

- *If c_D is sufficiently small compared to γ,*

$$\frac{c_D}{\gamma} \; < \; \mathrm{Prob}\,(NI|ND, s_F = T) \, ,$$

then both the cost of information acquisition in the market c_I and the noise in the market K

decrease disclosure:

$$\frac{\partial T}{\partial c_I} \; > \; 0 \, , \quad \frac{\partial T}{\partial K} \; > \; 0 \, .$$

As in the case of fixed information acquisition, the only assumption I impose in the corollary is that the cost of disclosure decreases disclosure. Even though it is intuitive, there exist equilibria where this assumption is violated. Corollary 4.1 holds in all equilibria in which the cost of disclosure decreases disclosure. In other equilibria, the results are reversed.

6 Optimal Information Acquisition in the Firm

Now let us study the setup in which the firm manager optimally chooses his information acquisition effort. In particular, let the manager choose the probability g of receiving a signal s_F before the trade (dotted red block in Figure 1). The choice of manager is unobservable and the effort is personally costly. Assume that the personal cost of effort is an increasing convex function

$$C_F(g) = \frac{c_F g^2}{2} \, ,$$

where c_F is the cost parameter. The firm manager's maximization problem is

$$\max_{g \in [0,\, 1]} \mathbb{E}[P|g, \hat{g}] - c_D g \operatorname{Prob}[s_F \geq T] - \frac{c_F g^2}{2},$$

where $\hat{g}$ is the market belief about the managerial effort. In the Perfect Bayesian equilibrium, beliefs coincide with optimal strategies, i.e. $g = \hat{g}$. The manager optimally takes into account that if he succeeds in acquiring the signal s_F, then he might still choose not to disclose it. I assume that there is no credible way for the manager to ex-ante commit to exert effort or to disclose.

If the value of managerial information γ is sufficiently low, then informed trading in the market crowds out both firm disclosure and information production in the firm. This result is intuitive. Indeed, if the value of managerial information γ is low, then informed trading reduces disclosure. This happens because the manager can free ride on informed traders who incorporate information about the fundamental firm value into the price. Consequently, informed trading crowds out the information production by the manager because the information is rarely disclosed.

Observe that the only real economic effect in the model comes through information acquisition by the firm. In particular, the ex-ante firm value is

$$\mathbb{E}[V] \;=\; \mathbb{E}[\theta] + \gamma \operatorname{Prob}(I) \;=\; \mathbb{E}[\theta] + \gamma g,$$

and it decreases whenever the firm manager exerts less effort g to collect his information. Consequently, informed trading decreases firm value if it crowds out information production in the firm.

The following proposition establishes the result.

Proposition 6.1 (Comparative Statics with Optimal Information Acquisition in the Firm)

Let the disclosure threshold increase in the cost of disclosure

$$\frac{\partial T}{\partial c_D} > 0,$$

and the optimal effort of the firm manager g decrease in the cost of effort

$$\frac{\partial g}{\partial c_F} < 0.$$

Also, let the value of managerial information γ be sufficiently small and let the technical condition

$$\frac{\partial}{\partial T}\left[\text{Prob}(\theta = 1|s_F \geq T) - \text{Prob}(\theta = 1|s_F = T)\right] < 0 \tag{1}$$

be satisfied.

Then

- *informed trading δ decreases disclosure, crowds out information production in the firm and decreases firm value:*
$$\frac{\partial T}{\partial \delta} > 0, \quad \frac{\partial g}{\partial \delta} < 0, \quad \frac{\partial \mathbb{E}[V]}{\partial \delta} < 0,$$

- *noise in the market K increases disclosure, crowds in information production in the firm and increases firm value:*
$$\frac{\partial T}{\partial K} < 0 \quad \frac{\partial g}{\partial K} > 0, \quad \frac{\partial \mathbb{E}[V]}{\partial \delta} > 0.$$

The conditions required to prove the result here are numerous, but they are mostly technical. In particular, I restrict the set of equilibria to the natural ones in which the cost of disclosure reduces disclosure and the cost of exerting effort reduces effort. The condition (1) is also intuitive. We can

see that the expression

$$\text{Prob}(\theta = 1 | s_F \geq T) - \text{Prob}(\theta = 1 | s_F = T)$$

is positive and shows how much more indicative of the large firm value are the signals larger than T as compared to the signal T. If the derivative of this expression with respect to T is negative, then the relative importance decreases as the signal T increases. For example, the condition (1) holds for normal signals with sufficiently large .

In contrast, the requirement of low γ is economically important. Informed trading decreases disclosure and information production in the firm only if it induces free-riding on informed traders. This only happens if γ is small as compared to the cost of disclosure c_D.
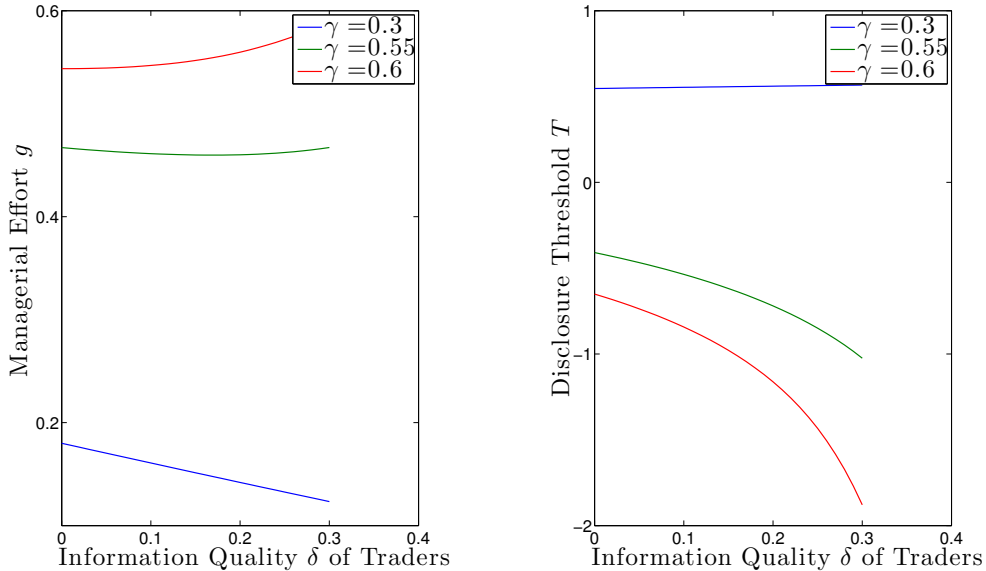

Figure 3: Managerial effort g and threshold T's dependence on information quality δ of traders. Parameters: $s_F |_\theta \sim \mathcal{N}(\theta, 1)$; $g = 0.3$; $\text{Prob}(\theta = 1) = 0.5$; $K = 1$; $c_D = 0.3$; $c_F = 0.5$

However, if γ is relatively high, then informed trading can crowd in information production in the firm. Indeed, if the value of managerial information γ is high, then informed trading increases disclosure. This happens because the manager cannot pool with uninformed managers and prefers to disclose to show that he is informed. Consequently, informed trading can crowd in information

production by the manager because the information is often disclosed. In other words, informed trading is an an ex-ante commitment device for the manager to disclose ex-post and thus increases his incentive to exert effort to collect information. Crowding in of information production in the firm leads to an increase in the firm value.

Figure 3 visually summarizes the findings. If the value of managerial information γ is low, then informed trading crowds out both information production in the firm and disclosure (blue line). If the value of managerial information γ is large, the effect can be reversed (red line) or non-monotonic (green line).

Figure 4 shows that the real effect can be positive, non-monotonic or negative depending on whether informed trading crowds in or crowds out information production in the firm.
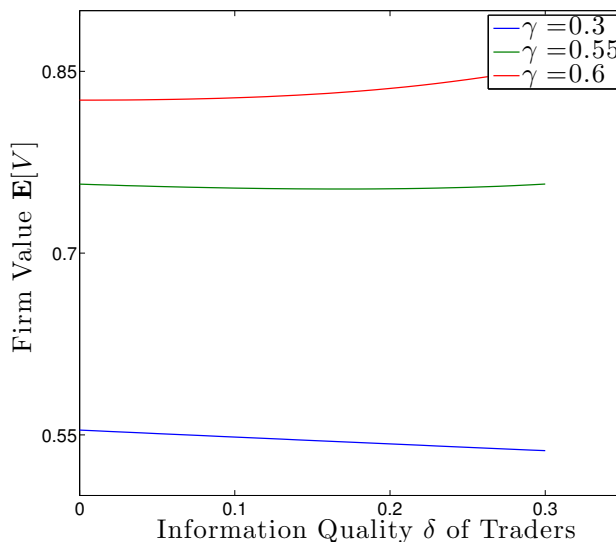

Figure 4: Real effect of informed trading.
Parameters: $s_F\big|_\theta \sim \mathcal{N}(\theta, 1)$; $g = 0.3$; $\mathrm{Prob}(\theta = 1) = 0.5$; $K = 1$; $c_D = 0.3$; $c_F = 0.5$

6.1   Optimal Information Acquisition in the Market and in the Firm

When both the information production in the firm and in the financial market are optimal, numerical results remain qualitatively the same. Figure 5 demonstrates the result: if information

acquisition in the financial market is easier, then information production in the firm can both go up and go down, depending on the relative strength of the effect of costly disclosure as compared to the effect of valuable information of the manager.
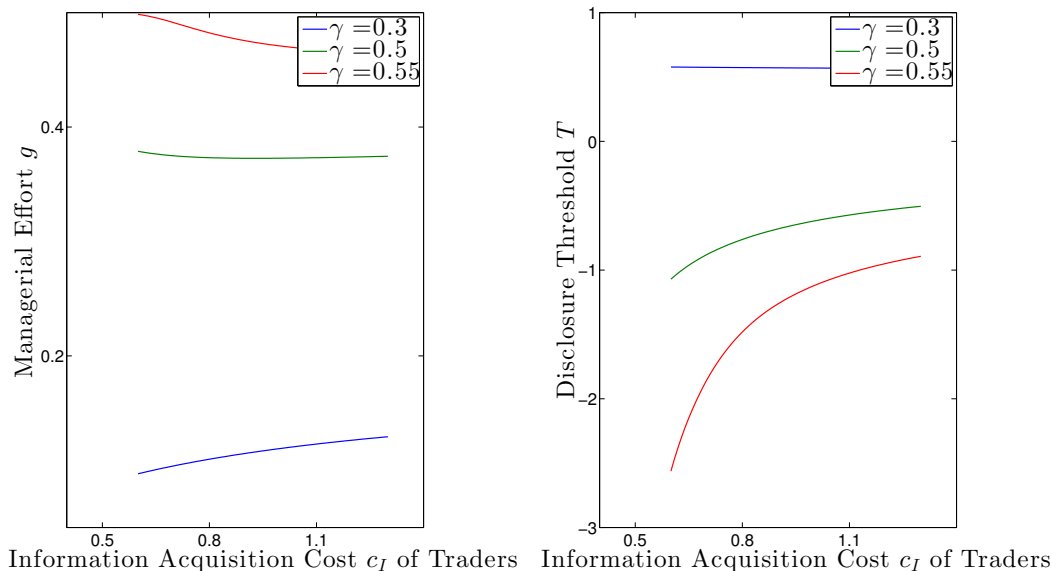

Figure 5: Managerial effort g and threshold T's dependence on the cost of information acquisition c_I of traders.
Parameters: $s_F\big|_\theta \sim \mathcal{N}(\theta, 1)$; $g = 0.3$; $\mathrm{Prob}(\theta = 1) = 0.5$; $K = 1$; $c_D = 0.3$; $c_F = 0.5$

7   Empirical Predictions

The comparative statics of the amount of voluntary disclosure with respect to informed trading provided in the paper depend on the relative importance of value of managerial information as compared to the cost of disclosure. Consequently, an empirical test of the results in the paper requires finding proxies for the informed trading, value of managerial information and the cost of disclosure. I discuss them below.

- Natural proxies for informed trading in the financial market of the firm's asset include probability of informed trading (pin) introduced by Easley et al. [1996], presence of institutional

traders (and particularly hedge funds), and the bid-ask spread.

- The value of managerial information should be larger for young firms, growth firms or firms with multiple investment opportunities as compared to old firms, value firms or firms with few investment opportunities.

- Following Verrecchia [1983], I suggest that the cost of disclosure should primarily include the cost of disclosing proprietary information to the firm's competitors. Consequently, it can be proxied by the degree of competition in the firm's product markets.

There is some empirical evidence about the effect of institutional holdings on the amount of voluntary disclosure. This evidence is mixed. For example, Ajinkya et al. [2005] find that institutional ownership increases issuance of management earnings forecasts. In contrast, Baik et al. [2014] find that hedge fund ownership decreases the frequency of such forecasts.

This paper suggests that these results can potentially suffer from an omitted variable problem. In particular, the model predicts that the effect of informed trading on the voluntary disclosure can be both positive and negative. The amount of voluntary disclosure should *increase* in the proxies for informed trading for relatively younger firms, growth firms or firms with multiple investment opportunities, as well as monopolistic firms. In contrast, the amount of voluntary disclosure should *decrease* in the proxies for informed trading for relatively older firms, value firms or firms with few investment opportunities, as well as the firms with many competitors.

The paper also suggests that informed trading is *detrimental for the firm value* if the value of managerial information is low. This is because informed trading crowds out information production in the firm. Consequently, we should observe empirically that these firms provide few voluntary disclosures and are traded at a discount. In contrast, informed trading can be *incremental for the firm value* if the value of managerial information is large. This is because informed trading can crowd in information production in the firm. Consequently, these firms may be traded at a

premium.

8   Conclusion

I study 'voluntary corporate disclosures to the financial market with informed traders who trade on their own signals about the fundamental value of the firm. The manager can exert costly effort to increase the chance to acquire his own information. Informed trading can both increase and decrease disclosure and crowd in and crowd out information acquisition in the firm. The direction of the effect depends on the relative importance of the value of firm manager's information compared to the cost of disclosure. The results hold for any distribution of signals of the manager and traders that satisfy the monotone likelihood ratio property. I provide a set of new empirical predictions to test the model.

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A Proofs

Proof. of Lemma 4.1

The market maker sets the price equal to the expected total value of the firm conditional on the total demand x and public information $\mathcal{F}_{public}$:

$$P = \mathbb{E}[\theta|x, \mathcal{F}_{public}] + \gamma \operatorname{Prob}[I|x, \mathcal{F}_{public}] .$$

Let us first calculate the first item and then the second term.

- The market maker observes the total demand

$$x = \int_0^1 x_i \, di + x_N = 1 - 2\Psi(S|\theta) + x_N .$$

The demand is uniformly distributed conditional on the firm value θ:

$$x\big|_\theta \sim [x_1(\theta), x_2(\theta)] ,$$

where
$$x_1(\theta) = 1 - 2\Psi(S|\theta) - K ,$$

$$x_2(\theta) = 1 - 2\Psi(S|\theta) + K .$$

By assumption, $\psi(s_i|\theta)$ satisfy monotone likelihood property. Also, by Conjecture 1, the threshold signal satisfies
$$\frac{\psi(S|\theta = 0)}{\psi(S|\theta = 1)} = 1 .$$

It follows that $\psi(s|\theta = 0) - \psi(s|\theta = 1) \geq 0$ for all $s \leq S$. Then,

$$\Psi(S|\theta = 0) - \Psi(S|\theta = 1) = \int_{-\infty}^{S} (\psi(s|\theta = 0) - \psi(s|\theta = 1)) \, ds \geq 0.$$

Thus, $x_1(\theta = 0) < x_1(\theta = 1)$ and $x_2(\theta = 1) > x_2(\theta = 0)$. Observing $x \leq x_1(\theta = 1) = \bar{x}$ reveals $\theta = 0$ and observing $x \geq x_2(\theta = 0) = \bar{x}$ reveals $\theta = 1$. However, when $x \in [\underline{x}, \bar{x}]$, the value of the total demand does not provide any additional information about θ to the market maker because the conditional densities of such x are the same and equal to $\frac{1}{2K}$ independently of θ. In this case, the expected value of θ from the point of view of the market maker is the expected value of the firm value fundamental θ given the disclosure or non-disclosure of the firm manager $\mathcal{F}_{public}$.

- Now let us calculate the second item. Observing $x \leq \underline{x}$ reveals $\theta = 0$ and thus

$$\text{Prob}\,[I|x \leq \underline{x}, \mathcal{F}_{public}] = \text{Prob}\,[I|\theta = 0, \mathcal{F}_{public}]\,.$$

Observing $x \geq \bar{x}$ reveals $\theta = 1$ and thus

$$\text{Prob}\,[I|x \geq \bar{x}, \mathcal{F}_{public}] = \text{Prob}\,[I|\theta = 1, \mathcal{F}_{public}]\,.$$

When $x \in [\underline{x}\,\bar{x}]$, the value of the total demand does not provide any additional information to the market maker. In this case,

$$\text{Prob}\,[I|x \in [\underline{x}\,\bar{x}], \mathcal{F}_{public}] = \text{Prob}\,[I|\mathcal{F}_{public}]\,.$$

∎

Proof. of Lemma 4.2

To see that δ indeed reflects the information quality, we can rewrite it in the following way:

$$\delta \;=\; 1 - \mathrm{Prob}(s_i \geq S|\theta = 0) - \mathrm{Prob}(s_i < S|\theta = 1)$$

$$=\; 1 - \mathrm{Prob}(x_i = 1|\theta = 0) - \mathrm{Prob}(x_i = -1|\theta = 1)\,.$$

Let us now show that $\delta < 1$. Since $\Psi(s_i|\theta) \in (0,\,1)$, one necessarily has that

$$\delta = \Psi(S|\theta = 0) - \Psi(S|\theta = 1) \;<\; 1\,.$$

One has that $\delta \to 1$ if $\Psi(S|\theta = 0) \to 1$ and $\Psi(S|\theta = 1) \to 0$. That means that with probability 1 all the signals s_i below the threshold S are observed in the low state $\theta = 0$ and signals above the threshold S are observed in the high state $\theta = 0$. In other words, the signal is perfectly revealing.

Let us now show that $\delta > 0$.

$$\frac{\partial \left(\Psi(s_i|\theta = 0) - \Psi(s_i|\theta = 1) \right)}{\partial s_i} = \psi(s_i|\theta = 0) - \psi(s_i|\theta = 1)\,.$$

Given the definition of S in Conjecture 1 and the monotone likelihood property, one has that

$$\delta \;=\; \max_{s_i} \Psi(s_i|\theta = 0) - \Psi(s_i|\theta = 1) \;=\; \Psi(S|\theta = 0) - \Psi(S|\theta = 1)$$

As a maximum,

$$\delta \;>\; \Psi(-\infty|\theta = 0) - \Psi(-\infty|\theta = 1) \;=\; 0\,.$$

One has that $\delta \to 0$ if $\Psi(S|\theta = 0) = \Psi(S|\theta = 1)$. That means that probability to observe the signals below the threshold S are the same in the low state $\theta = 0$ and the high state $\theta = 1$. In other words, the signal is a pure noise. ∎

Proof. of Lemma 4.3

A risk-neutral trader i's portfolio problem is:

$$\max_{x_i \in [-1,1]} \mathbb{E}\left(x_i(V - P)|s_i, \mathcal{F}_{public}\right).$$

The trader i's optimal portfolio is

$$x_i = \begin{cases} 1, & \mathbb{E}\left[V - P|s_i, \mathcal{F}_{public}\right] \geq 0, \\[2ex] -1, & \mathbb{E}\left[V - P|s_i, \mathcal{F}_{public}\right] < 0. \end{cases}$$

Thus, the trader buys as many assets as he can ($x_i = 1$) if and only if

$$\mathbb{E}\left[V - P|s_i, \mathcal{F}_{public}\right] \geq 0$$

and short sells otherwise. The price P is given by Lemma 4.1 and we have

$\mathbb{E}\left[P|s_i, \mathcal{F}_{public}\right]$

$= \mathbb{E}\left[P|\theta = 1, s_i, \mathcal{F}_{public}\right] \text{Prob}(\theta = 1|s_i, \mathcal{F}_{public}) + \mathbb{E}\left[P|\theta = 0, s_i, \mathcal{F}_{public}\right] \text{Prob}(\theta = 0|s_i, \mathcal{F}_{public})$

$= \mathbb{E}\left[P|\theta = 1, \mathcal{F}_{public}\right] \text{Prob}(\theta = 1|s_i, \mathcal{F}_{public}) + \mathbb{E}\left[P|\theta = 0, \mathcal{F}_{public}\right] \text{Prob}(\theta = 0|s_i, \mathcal{F}_{public})$

$= \Big(\left[\mathbb{E}\left[\theta|\mathcal{F}_{public}\right] + \gamma \text{Prob}[I|\mathcal{F}_{public}]\right] \text{Prob}(x \in (\underline{x}, \bar{x})|\theta = 1)$

$\quad + \left[1 + \gamma \text{Prob}[I|\theta = 1, \mathcal{F}_{public}]\right] \text{Prob}(x \geq \bar{x}|\theta = 1) \Big) \text{Prob}(\theta = 1|s_i, \mathcal{F}_{public})$

$+ \Big(\left[\mathbb{E}\left[\theta|\mathcal{F}_{public}\right] + \gamma \text{Prob}[I|\mathcal{F}_{public}]\right] \text{Prob}(x \in (\underline{x}, \bar{x})|\theta = 0)$

$\quad + \left[0 + \gamma \text{Prob}[I|\theta = 0, \mathcal{F}_{public}]\right] \text{Prob}(x \leq \underline{x}|\theta = 0) \Big) \text{Prob}(\theta = 0|s_i, \mathcal{F}_{public}).$

Now, denote by $\delta = \Psi(S|\theta = 0) - \Psi(S|\theta = 1)$ and observe that

$$
\begin{aligned}
\text{Prob}(x \in (\underline{x}, \bar{x})|\theta = 1) &= \frac{2K - 2\delta}{2K} = 1 - \frac{\delta}{K}, \\
\text{Prob}(x \geq \bar{x}|\theta = 1) &= \frac{\delta}{K}, \\
\text{Prob}(x \in (\underline{x}, \bar{x})|\theta = 0) &= 1 - \frac{\delta}{K}, \\
\text{Prob}(x \leq \underline{x}|\theta = 0) &= \frac{\delta}{K},
\end{aligned}
$$

Substituting these probabilities in the expression above, one has

$$
\begin{aligned}
\mathbb{E}\left[P|s_i, \mathcal{F}_{public}\right] &= \left(\left[\mathbb{E}\left[\theta|\mathcal{F}_{public}\right] + \gamma \text{Prob}[I|\mathcal{F}_{public}]\right]\left(1 - \frac{\delta}{K}\right)\right. \\
&\quad + \left.\left[1 + \gamma \text{Prob}[I|\theta = 1, \mathcal{F}_{public}]\right]\frac{\delta}{K}\right) \text{Prob}(\theta = 1|s_i, \mathcal{F}_{public}) \\
&\quad + \left(\left[\mathbb{E}\left[\theta|\mathcal{F}_{public}\right] + \gamma \text{Prob}[I|\mathcal{F}_{public}]\right]\left(1 - \frac{\delta}{K}\right)\right. \\
&\quad + \left.\left[0 + \gamma \text{Prob}[I|\theta = 0, \mathcal{F}_{public}]\right]\frac{\delta}{K}\right) \text{Prob}(\theta = 0|s_i, \mathcal{F}_{public}) \\
&= \left(\mathbb{E}\left[\theta|\mathcal{F}_{public}\right] + \gamma \text{Prob}[I|\mathcal{F}_{public}]\right)\left(1 - \frac{\delta}{K}\right) \\
&\quad + \left(\mathbb{E}[\theta|s_i, \mathcal{F}_{public}] + \gamma \text{Prob}[I|s_i, \mathcal{F}_{public}]\right)\frac{\delta}{K}
\end{aligned}
$$

Recall that the trader i buys if and only if

$$
\mathbb{E}\left[V - P|s_i, \mathcal{F}_{public}\right] \geq 0
$$

or, equivalently,

$$
\mathbb{E}\left[\theta - P|s_i, \mathcal{F}_{public}\right] + \gamma \text{Prob}[I|s_i, \mathcal{F}_{public}] \geq 0
$$

or, equivalently,

$$
\left(\mathbb{E}\left[\theta|s_i, \mathcal{F}_{public}\right] - \mathbb{E}\left[\theta|\mathcal{F}_{public}\right] + \gamma\left(\text{Prob}[I|s_i, \mathcal{F}_{public}] - \text{Prob}[I|\mathcal{F}_{public}]\right)\right)\left(1 - \frac{\delta}{K}\right) \geq 0.
$$

I show in Lemma 4.2 that $\delta \in (0, 1)$ and by assumption $K > 1$, so that this is equivalent to

$$\mathbb{E}\left[\theta|s_i, \mathcal{F}_{public}\right] - \mathbb{E}\left[\theta|\mathcal{F}_{public}\right] + \gamma \left(\text{Prob}[I|s_i, \mathcal{F}_{public}] - \text{Prob}[I|\mathcal{F}_{public}]\right) \geq 0\,.$$

The expression can be rewritten as

$$\text{Prob}\left(\theta = 1|s_i, \mathcal{F}_{public}\right) - \text{Prob}\left(\theta = 1|\mathcal{F}_{public}\right) + \gamma \left(\text{Prob}[I|s_i, \mathcal{F}_{public}] - \text{Prob}[I|\mathcal{F}_{public}]\right) \geq 0\,,$$

or

$$\text{Prob}\left(\theta = 1|s_i, \mathcal{F}_{public}\right) - \text{Prob}\left(\theta = 1|\mathcal{F}_{public}\right)$$
$$+ \gamma \left(\text{Prob}[I|\theta = 1, \mathcal{F}_{public}]\,\text{Prob}\left(\theta = 1|s_i, \mathcal{F}_{public}\right) + \text{Prob}[I|\theta = 0, \mathcal{F}_{public}]\left(1 - \text{Prob}\left(\theta = 1|s_i, \mathcal{F}_{public}\right)\right)\right)$$
$$- \gamma \left(\text{Prob}[I|\theta = 1, \mathcal{F}_{public}]\,\text{Prob}\left(\theta = 1|\mathcal{F}_{public}\right) + \text{Prob}[I|\theta = 0, \mathcal{F}_{public}]\left(1 - \text{Prob}\left(\theta = 1|\mathcal{F}_{public}\right)\right)\right) \geq 0\,,$$

or, equivalently,

$$\left(\text{Prob}\left(\theta = 1|s_i, \mathcal{F}_{public}\right) - \text{Prob}\left(\theta = 1|\mathcal{F}_{public}\right)\right)\left(1 - \gamma \left(\text{Prob}[I|\theta = 0, \mathcal{F}_{public}] - \text{Prob}[I|\theta = 1, \mathcal{F}_{public}]\right)\right) \geq 0\,,$$

When $\mathcal{F}_{public} = s_F$, the second factor is equal to 1 because $\text{Prob}[I|\theta = 0, \mathcal{F}_{public}] = \text{Prob}[I|\theta = 1, \mathcal{F}_{public}] = 1$. When $\mathcal{F}_{public} = ND$, the second factor is positive if $\gamma \leq 1$. It follows that a trader with a signal s_i buys the asset if and only if

$$\text{Prob}\left(\theta = 1|s_i, \mathcal{F}_{public}\right) - \text{Prob}\left(\theta = 1|\mathcal{F}_{public}\right) \geq 0,$$

or, by the Bayes law,

$$\frac{1}{1 + \frac{\text{Prob}\left(\theta=0|\mathcal{F}_{public}\right)}{\text{Prob}\left(\theta=1|\mathcal{F}_{public}\right)} \frac{\text{Prob}(s_i|\theta=0)}{\text{Prob}(s_i|\theta=1)}} - \frac{1}{1 + \frac{\text{Prob}\left(\theta=0|\mathcal{F}_{public}\right)}{\text{Prob}\left(\theta=1|\mathcal{F}_{public}\right)}} \geq 0$$

which holds if and only if

$$\frac{\text{Prob}\left(s_i|\theta=0\right)}{\text{Prob}\left(s_i|\theta=1\right)} \leq 1$$

or

$$\frac{\psi(s_i|\theta=0)}{\psi(s_i|\theta=1)} \leq 1.$$

Given the monotone likelihood ratio property imposed on ψ, the optimal strategy is indeed of a threshold type, such that the threshold S satisfies

$$\frac{\psi(S|\theta=0)}{\psi(S|\theta=1)} = 1.$$

∎

Proof. of Proposition 4.1

We can rewrite the expression for $\mathbb{E}\left[P|s_F, \mathcal{F}_{public}\right]$ given the price in Lemma 4.1 as follows:

$$
\begin{aligned}
\mathbb{E}\left[P|s_F, \mathcal{F}_{public}\right] \;=\; & \left(\left[\mathbb{E}\left[\theta|\mathcal{F}_{public}\right] + \gamma\,\mathrm{Prob}[I|\mathcal{F}_{public}]\right]\left(1 - \frac{\delta}{K}\right)\right. \\
& \left. + \;[1 + \gamma\,\mathrm{Prob}[I|\theta = 1, \mathcal{F}_{public}]]\,\frac{\delta}{K}\right)\mathrm{Prob}(\theta = 1|s_F) \\
& + \left(\left[\mathbb{E}\left[\theta|\mathcal{F}_{public}\right] + \gamma\,\mathrm{Prob}[I|\mathcal{F}_{public}]\right]\left(1 - \frac{\delta}{K}\right)\right. \\
& \left. + \;[0 + \gamma\,\mathrm{Prob}[I|\theta = 0, \mathcal{F}_{public}]]\,\frac{\delta}{K}\right)\mathrm{Prob}(\theta = 0|s_F) \\
\;=\; & \left(\mathbb{E}\left[\theta|\mathcal{F}_{public}\right] + \gamma\,\mathrm{Prob}[I|\mathcal{F}_{public}]\right)\left(1 - \frac{\delta}{K}\right) \\
& + \left(\mathbb{E}[\theta|s_F] + \gamma\big(\mathrm{Prob}[I|\theta = 1, \mathcal{F}_{public}]\,\mathrm{Prob}\left(\theta = 1|s_F\right)\right. \\
& \left. + \,\mathrm{Prob}[I|\theta = 0, \mathcal{F}_{public}]\,\mathrm{Prob}(\theta = 0|s_F)\big)\right)\frac{\delta}{K}\,.
\end{aligned}
$$

We can further simplify the expression as

$$
\begin{aligned}
\mathbb{E}\left[P|s_F, \mathcal{F}_{public}\right] \;=\; & \mathbb{E}\left[\theta|\mathcal{F}_{public}\right]\left(1 - \frac{\delta}{K}\right) + \frac{\delta}{K}\mathbb{E}[\theta|s_F] \\
& + \;\gamma\left(\mathrm{Prob}[I|\mathcal{F}_{public}]\left(1 - \frac{\delta}{K}\right)\right. \\
& \left. + \;\big(\mathrm{Prob}[I|\theta = 1, \mathcal{F}_{public}]\,\mathrm{Prob}\left(\theta = 1|s_F\right) + \mathrm{Prob}[I|\theta = 0, \mathcal{F}_{public}]\,\mathrm{Prob}(\theta = 0|s_F)\big)\frac{\delta}{K}\right)\,.
\end{aligned}
$$

If $\mathcal{F}_{public} = s_F$, then

$$
\begin{aligned}
\mathbb{E}\left[P|s_F, \mathcal{F}_{public} = s_F\right] \;=\; & \mathbb{E}\left[\theta|s_F\right]\left(1 - \frac{\delta}{K}\right) + \frac{\delta}{K}\mathbb{E}[\theta|s_F] + \gamma \\
\;=\; & \mathbb{E}[\theta|s_F] + \gamma\,.
\end{aligned}
$$

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If $\mathcal{F}_{public} = ND$, then

$$\mathbb{E}\left[P|s_F, \mathcal{F}_{public} = ND\right] = \mathbb{E}\left[\theta|ND\right]\left(1 - \frac{\delta}{K}\right) + \frac{\delta}{K}\mathbb{E}[\theta|ND]$$
$$+ \gamma\left(\text{Prob}[I|ND]\left(1 - \frac{\delta}{K}\right) + \text{Prob}[I|ND, s_F]\frac{\delta}{K}\right).$$

Now, we have that the firm manager discloses if and only if

$$\mathbb{E}[P|s_F, \mathcal{F}_{public} = s_F] - c_D \geq \mathbb{E}[P|s_F, \mathcal{F}_{public} = ND],$$

or, equivalently, if

$$\mathbb{E}(\theta|\mathcal{F}_{public} = s_F) \geq \mathbb{E}(\theta|\mathcal{F}_{public} = ND) + \frac{c_D}{1 - \frac{\delta}{K}} - \xi,$$

where

$$\xi = \gamma\left(\frac{1 - \frac{\delta}{K}\text{Prob}\left(I|ND, s_F\right)}{1 - \frac{\delta}{K}} - \text{Prob}\left(I|ND\right)\right) \geq 0. \tag{2}$$

If g is sufficiently close to 0 or 1 or γ or $\frac{\delta}{K}$ are sufficiently small, all equilibria are of a threshold type. I prove this by showing that

$$\frac{\partial}{\partial s_F}\left(\mathbb{E}(\theta|\mathcal{F}_{public} = s_F) + \xi\right) > 0,$$

so that the firm manager is always more willing to disclose higher signals independently of the beliefs of the market in the case of no disclosure.

We have

$$\mathbb{E}(\theta|\mathcal{F}_{public} = s_F) = \frac{1}{1 + \frac{\text{Prob}(\theta=0)}{\text{Prob}(\theta=1)}\frac{f(s_F|\theta=0)}{f(s_F|\theta=1)}}$$

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and

$$\frac{\partial}{\partial s_F}\mathbb{E}(\theta|\mathcal{F}_{public}=s_F) = \frac{-\frac{\text{Prob}(\theta=0)}{\text{Prob}(\theta=1)}}{\left(1+\frac{\text{Prob}(\theta=0)}{\text{Prob}(\theta=1)}\frac{f(s_F|\theta=0)}{f(s_F|\theta=1)}\right)^2}\frac{\partial}{\partial s_F}\left(\frac{f(s_F|\theta=0)}{f(s_F|\theta=1)}\right).$$

Also,

$$\text{Prob}\,(I|ND,s_F) = \frac{\text{Prob}\,(I,ND,s_F)}{\text{Prob}\,(I,ND,s_F)+\text{Prob}\,(NI,ND,s_F)}$$

$$= \frac{1}{1+\frac{1-g}{g}\frac{1+\frac{\text{Prob}(\theta=0)}{\text{Prob}(\theta=1)}\frac{f(s_F|\theta=0)}{f(s_F|\theta=1)}}{\text{Prob}(ND|\theta=1)+\text{Prob}(ND|\theta=0)\frac{\text{Prob}(\theta=0)}{\text{Prob}(\theta=1)}\frac{f(s_F|\theta=0)}{f(s_F|\theta=1)})}}$$

and

$$\frac{\partial}{\partial s_F}\,\text{Prob}\,(I|ND,s_F) =$$

$$\frac{g(1-g)\frac{\text{Prob}(\theta=0)}{\text{Prob}(\theta=1)}\left(\text{Prob}(ND|\theta=0)-\text{Prob}(ND|\theta=1)\right)\frac{\partial}{\partial s_F}\left(\frac{f(s_F|\theta=0)}{f(s_F|\theta=1)}\right)}{\left[g\left(\text{Prob}(ND|\theta=1)+\text{Prob}(ND|\theta=0)\frac{\text{Prob}(\theta=0)}{\text{Prob}(\theta=1)}\frac{f(s_F|\theta=0)}{f(s_F|\theta=1)}\right)+(1-g)\left(1+\frac{\text{Prob}(\theta=0)}{\text{Prob}(\theta=1)}\frac{f(s_F|\theta=0)}{f(s_F|\theta=1)}\right)\right]^2}.$$

The derivative is thus

$$\frac{\partial}{\partial s_F}\left(\mathbb{E}(\theta|\mathcal{F}_{public}=s_F)+\xi\right)$$

$$= \frac{\partial}{\partial s_F}\mathbb{E}(\theta|\mathcal{F}_{public}=s_F)-\frac{\gamma\frac{\delta}{K}}{1-\frac{\delta}{K}}\frac{\partial}{\partial s_F}\,\text{Prob}\,(I|ND,s_F).$$

The first item is positive because by the monotone likelihood property $\frac{\partial}{\partial s_F}\left(\frac{f(s_F|\theta=0)}{f(s_F|\theta=1)}\right) < 0$. The second item can be made sufficiently small if g is sufficiently close to 0 or 1 or γ or $\frac{\delta}{K}$ are sufficiently small. This finishes the proof.

∎

Proof. of Corollary 4.1

Proposition 4.1 shows that any equilibrium is of a threshold type. The equilibrium value of a threshold T solves

$$\mathbb{E}(\theta|\mathcal{F}_{public} = ND) + \frac{c_D}{1 - \frac{\delta}{K}} - \xi - \mathbb{E}(\theta|\mathcal{F}_{public} = T) = 0.$$

Denote the left-hand side of the equation by $\mathcal{L}$. Now we can compute the derivatives in the corollary as implicit function derivatives. In particular,

$$\frac{\partial T}{\partial c_D} = -\frac{\frac{\partial \mathcal{L}}{\partial c_D}}{\frac{\partial \mathcal{L}}{\partial T}},$$

$$\frac{\partial T}{\partial \delta} = -\frac{\frac{\partial \mathcal{L}}{\partial \delta}}{\frac{\partial \mathcal{L}}{\partial T}},$$

$$\frac{\partial T}{\partial K} = -\frac{\frac{\partial \mathcal{L}}{\partial K}}{\frac{\partial \mathcal{L}}{\partial T}}.$$

To compare the signs of derivatives we need to compare the signs of the numerators of the ratios:

$$\frac{\partial \mathcal{L}}{\partial c_D} = \frac{1}{1 - \frac{\delta}{K}} > 0,$$

$$\frac{\partial \mathcal{L}}{\partial \frac{\delta}{K}} = \frac{c_D - \gamma \left(1 - \text{Prob}\left(I | ND, s_F\right)\right)}{\left(1 - \frac{\delta}{K}\right)^2}.$$

This finishes the proof. ∎

Proof. of Lemma 5.1

We have that

$$
\mathbb{E}[\text{Profit}_i \,|\mathcal{F}_{public}]
$$

$$
= \ \mathbb{E}[\text{Profit}_i \,|\theta = 1, \, I, \, \mathcal{F}_{public}] \, \text{Prob}(\theta = 1, \, I|\mathcal{F}_{public})
$$

$$
+ \ \mathbb{E}[\text{Profit}_i \,|\theta = 0, \, I, \, \mathcal{F}_{public}] \, \text{Prob}(\theta = 0, \, I|\mathcal{F}_{public}) \tag{3}
$$

$$
+ \ \mathbb{E}[\text{Profit}_i \,|\theta = 1, \, NI, \, \mathcal{F}_{public}] \, \text{Prob}(\theta = 1, \, NI|\mathcal{F}_{public})
$$

$$
+ \ \mathbb{E}[\text{Profit}_i \,|\theta = 0, \, NI, \, \mathcal{F}_{public}] \, \text{Prob}(\theta = 0, \, NI|\mathcal{F}_{public}) \,.
$$

Now we can write out the first factors in each of these four items:

$$
\mathbb{E}[\text{Profit}_i \,|\theta = 1, \, I, \, \mathcal{F}_{public}]
$$

$$
= \ \left[\left(1 - \frac{\delta^*}{K} \right) \left(1 - \mathbb{E}[\theta|\mathcal{F}_{public}] + \gamma \left(1 - \text{Prob}(I|\mathcal{F}_{public}) \right) \right) \right.
$$

$$
\left. + \frac{\delta^*}{K} \gamma \left(1 - \text{Prob}(I|\mathcal{F}_{public}, \theta = 1) \right) \right] \left(1 - 2\Psi(S|\theta = 1) \right) \,,
$$

$$
\mathbb{E}[\text{Profit}_i \,|\theta = 0, \, I, \, \mathcal{F}_{public}]
$$

$$
= \ \left[\left(1 - \frac{\delta^*}{K} \right) \left(-\mathbb{E}[\theta|\mathcal{F}_{public}] + \gamma \left(1 - \text{Prob}(I|\mathcal{F}_{public}) \right) \right) \right.
$$

$$
\left. + \frac{\delta^*}{K} \gamma \left(1 - \text{Prob}(I|\mathcal{F}_{public}, \theta = 0) \right) \right] \left(1 - 2\Psi(S|\theta = 0) \right) \,,
$$

$$
\mathbb{E}[\text{Profit}_i \,|\theta = 1, \, NI, \, \mathcal{F}_{public}]
$$

$$
= \ \left[\left(1 - \frac{\delta^*}{K} \right) \left(1 - \mathbb{E}[\theta|\mathcal{F}_{public}] - \gamma \, \text{Prob}(I|\mathcal{F}_{public}) \right) \right.
$$

$$
\left. - \frac{\delta^*}{K} \gamma \, \text{Prob}(I|\mathcal{F}_{public}, \theta = 1) \right] \left(1 - 2\Psi(S|\theta = 1) \right) \,,
$$

$$
\mathbb{E}[\text{Profit}_i \,|\theta = 0, \, NI, \, \mathcal{F}_{public}]
$$

$$
= \ \left[\left(1 - \frac{\delta^*}{K} \right) \left(-\mathbb{E}[\theta|\mathcal{F}_{public}] - \gamma \, \text{Prob}(I|\mathcal{F}_{public}) \right) \right.
$$

$$
\left. - \frac{\delta^*}{K} \gamma \, \text{Prob}(I|\mathcal{F}_{public}, \theta = 0) \right] \left(1 - 2\Psi(S|\theta = 0) \right) \,.
$$

Substituting these in 3, one obtains

$$\mathbb{E}[\text{Profit}_i \,|\mathcal{F}_{public}] =$$

$$2\left(1 - \frac{\delta^*}{K}\right)\text{Prob}(\theta = 1|\mathcal{F}_{public})\,\text{Prob}(\theta = 0|\mathcal{F}_{public})\,(\Psi(S|\theta = 0) - \Psi(S|\theta = 1))$$

$$+\ \gamma\left[\text{Prob}(\theta = 1, I|\mathcal{F}_{public}) - \left(1 - \frac{\delta^*}{K}\right)\text{Prob}(I|\mathcal{F}_{public})\,\text{Prob}(\theta = 1|\mathcal{F}_{public})\right.$$

$$\left. -\ \frac{\delta^*}{K}\text{Prob}(\theta = 1, I|\mathcal{F})_{public}\right](1 - 2\Psi(S|\theta = 1))$$

$$+\ \gamma\left[\text{Prob}(\theta = 0, I|\mathcal{F}_{public}) - \left(1 - \frac{\delta^*}{K}\right)\text{Prob}(I|\mathcal{F}_{public})\,\text{Prob}(\theta = 0|\mathcal{F}_{public})\right.$$

$$\left. -\ \frac{\delta^*}{K}\text{Prob}(\theta = 0, I|\mathcal{F})_{public}\right](1 - 2\Psi(S|\theta = 0))\ .$$

Recalling that

$$\delta = (\Psi(S|\theta = 0) - \Psi(S|\theta = 1))\ ,$$

one can further simplify this to

$$\mathbb{E}[\text{Profit}_i \,|\mathcal{F}_{public}]$$

$$=\ 2\delta\left(1 - \frac{\delta^*}{K}\right)\text{Prob}(\theta = 1|\mathcal{F}_{public})\,\text{Prob}(\theta = 0|\mathcal{F}_{public})\big[1 - \gamma\,(\text{Prob}(I|\theta = 0, \mathcal{F}_{public}) - \text{Prob}(I|\theta = 1, \mathcal{F}_{public}))\big]\ ,$$

and the last thing to observe is that

$$\text{Prob}(\theta = 1|\mathcal{F}_{public})\,\text{Prob}(\theta = 0|\mathcal{F}_{public})\ =\ \mathbb{V}\text{ar}(\theta|\mathcal{F}_{public})$$

because θ is a Bernoulli random variable. This finishes the proof. ∎

Proof. of Lemma 5.2

A trader i chooses information quality δ to maximise

$$\max_{\delta \in [0,1]} \; \mathbb{E}[\text{Profit}_i \,|\mathcal{F}_{public}] - \frac{c_I \delta^2}{2} \,.$$

given his belief that all other traders choose information quality δ^*. According to Lemma 5.1,

$$\mathbb{E}[\text{Profit}_i \,|\mathcal{F}_{public}] \; = \; 2\delta \left(1 - \frac{\delta^*}{K}\right) \mathbb{V}\text{ar}(\theta|\mathcal{F}_{public})\Big[1 - \gamma \left(\text{Prob}(I|\theta = 0, \mathcal{F}_{public}) - \text{Prob}(I|\theta = 1, \mathcal{F}_{public})\right)\Big] \,.$$

The first order condition with respect to δ gives

$$2 \left(1 - \frac{\delta^*}{K}\right) \mathbb{V}\text{ar}(\theta|\mathcal{F}_{public})\Big[1 - \gamma \left(\text{Prob}(I|\theta = 0, \mathcal{F}_{public}) - \text{Prob}(I|\theta = 1, \mathcal{F}_{public})\right)\Big] - c_I \delta \; = \; 0 \,.$$

In a symmetric equilibrium,

$$\delta \; = \; \delta^* \; = \; \frac{1}{\dfrac{1}{K} + \dfrac{c_I}{2\mathbb{V}\text{ar}(\theta|\mathcal{F}_{public})\Big[1 - \gamma\big(\text{Prob}(I|\theta=0,\mathcal{F}_{public}) - \text{Prob}(I|\theta=1,\mathcal{F}_{public})\big)\Big]}} \,.$$

∎

Proof. of Proposition 5.1

The proof is the same as in Proposition 4.1 except that now the information quality δ is different when there is disclosure $\delta(\mathcal{F}_{public} = s_F)$ and when there is no disclosure $\delta(\mathcal{F}_{public} = ND)$. Substituting $\delta(\mathcal{F}_{public} = ND)$ from Lemma 5.2 finishes the proof. The proof that all equilibria are of a threshold type is similar to the one in Proposition 4.1 ∎

Proof. of Corollary 5.1

Proposition 5.1 shows that any equilibrium is of a threshold type. The equilibrium value of a threshold T solves

$$- \mathbb{E}(\theta|\mathcal{F}_{public} = T) + \mathbb{E}(\theta|\mathcal{F}_{public} = ND) + \left(c_D - \gamma \operatorname{Prob}(NI|ND, T)\right)\left(1 + \frac{2\mathbb{V}\mathrm{ar}(\theta|ND)M(\gamma, ND)}{c_I K}\right)$$

$$- \gamma\left[\operatorname{Prob}(NI|ND) - \operatorname{Prob}(NI|ND, T)\right] = 0.$$

Denote the left-hand side of the equation by $\mathcal{L}$. Now we can compute the derivatives in the corollary as implicit function derivatives. In particular,

$$\frac{\partial T}{\partial c_D} = -\frac{\frac{\partial \mathcal{L}}{\partial c_D}}{\frac{\partial \mathcal{L}}{\partial T}},$$

$$\frac{\partial T}{\partial c_I} = -\frac{\frac{\partial \mathcal{L}}{\partial c_I}}{\frac{\partial \mathcal{L}}{\partial T}},$$

$$\frac{\partial T}{\partial K} = -\frac{\frac{\partial \mathcal{L}}{\partial K}}{\frac{\partial \mathcal{L}}{\partial T}}.$$

To compare the signs of derivatives we need to compare the signs of the numerators of the ratios:

$$\frac{\partial \mathcal{L}}{\partial c_D} = 1 + \frac{2\mathbb{V}\mathrm{ar}(\theta|ND)M(\gamma, ND)}{c_I K} > 0,$$

$$\frac{\partial \mathcal{L}}{\partial c_I} = -\left(c_D - \gamma \operatorname{Prob}(NI|ND, T)\right)\frac{2\mathbb{V}\mathrm{ar}(\theta|ND)M(\gamma, ND)}{c_I^2 K},$$

$$\frac{\partial \mathcal{L}}{\partial K} = -\left(c_D - \gamma \operatorname{Prob}(NI|ND, T)\right)\frac{2\mathbb{V}\mathrm{ar}(\theta|ND)M(\gamma, ND)}{c_I K^2}.$$

∎

Proof. of Proposition 6.1 Let the market maker and traders believe that the effort that the firm manager exerts to acquire information is g and the threshold of disclosure is T. Let us now solve the ex-ante maximization program of the manager who chooses his level of effort $\bar{g}$:

$$\max_{\bar{g} \in [0,1]} \mathbb{E}[P|\bar{g}] - c_D \bar{g} \operatorname{Prob}(s_F \geq T) - \frac{c_F \bar{g}^2}{2} .$$

The first order condition with respect to the optimal level of effort $\bar{g}$ is

$$\frac{\partial \mathbb{E}[P|\bar{g}]}{\partial \bar{g}} - c_D \operatorname{Prob}(s_F \geq T) - c_F \bar{g} = 0 .$$

Let us first compute $\mathbb{E}[P|\bar{g}]$ and then we can compute it's derivative:

$$\begin{aligned}
\mathbb{E}[P|\bar{g}] &= \operatorname{Prob}(\theta = 0)\left(\bar{g} \operatorname{Prob}(s_F \leq T|\theta = 0) + 1 - \bar{g}\right) \mathbb{E}[P|ND, \theta = 0] \\
&\quad + \operatorname{Prob}(\theta = 0)\bar{g} \int_{s_F \geq T} \mathbb{E}[P|s_F, \theta = 0] f(s_F|0)\, ds_F \\
&\quad + \operatorname{Prob}(\theta = 1)\left(\bar{g} \operatorname{Prob}(s_F \leq T|\theta = 1) + 1 - \bar{g}\right) \mathbb{E}[P|ND, \theta = 1] \\
&\quad + \operatorname{Prob}(\theta = 1)\bar{g} \int_{s_F \geq T} \mathbb{E}[P|s_F, \theta = 1] f(s_F|1)\, ds_F .
\end{aligned}$$

We can write out the expected prices in each of the four items in the expression above, so that

we have:

$$
\begin{aligned}
\mathbb{E}[P|\bar{g}] \;=\; & \text{Prob}(\theta = 0)\,(\bar{g}\,\text{Prob}(s_F \leq T|\theta = 0) + 1 - \bar{g}) \\
& \left[\left(1 - \frac{\delta}{K}\right)(\mathbb{E}[\theta|ND] + \gamma\,\text{Prob}(I|ND)) + \frac{\delta}{K}(0 + \gamma\,\text{Prob}(I|ND, \theta = 0)) \right] \\
& +\ \text{Prob}(\theta = 0)\bar{g} \\
& \int_{s_F \geq T} \left[\left(1 - \frac{\delta}{K}\right)(\mathbb{E}[\theta|s_F] + \gamma) + \frac{\delta}{K}(0 + \gamma) \right] f(s_F|0)\,ds_F \\
& +\ \text{Prob}(\theta = 1)\,(\bar{g}\,\text{Prob}(s_F \leq T|\theta = 1) + 1 - \bar{g}) \\
& \left[\left(1 - \frac{\delta}{K}\right)(\mathbb{E}[\theta|ND] + \gamma\,\text{Prob}(I|ND)) + \frac{\delta}{K}(1 + \gamma\,\text{Prob}(I|ND, \theta = 1)) \right] \\
& +\ \text{Prob}(\theta = 1)\bar{g} \\
& \int_{s_F \geq T} \left[\left(1 - \frac{\delta}{K}\right)(\mathbb{E}[\theta|s_F] + \gamma) + \frac{\delta}{K}(1 + \gamma) \right] f(s_F|1)\,ds_F.
\end{aligned}
$$

Observe that the only terms containing the effort $\bar{g}$ are the first lines in each of the four terms. All second lines contain g, which is the belief of traders and the market maker about the level of effort.

Now we can compute the derivative $\frac{\partial \mathbb{E}[P|\bar{g}]}{\partial \bar{g}}$ for the first order condition. We have

$$
\begin{aligned}
\frac{\partial \mathbb{E}[P|\bar{g}]}{\partial \bar{g}} \;=\; & -\text{Prob}(\theta = 0)\,\text{Prob}(s_F \geq T|\theta = 0) \\
& \left[\left(1 - \frac{\delta}{K}\right)(\mathbb{E}[\theta|ND] + \gamma\,\text{Prob}(I|ND)) + \frac{\delta}{K}(0 + \gamma\,\text{Prob}(I|ND, \theta = 0)) \right] \\
& +\ \text{Prob}(\theta = 0) \int_{s_F \geq T} \left[\left(1 - \frac{\delta}{K}\right)(\mathbb{E}[\theta|s_F] + \gamma) + \frac{\delta}{K}(0 + \gamma) \right] f(s_F|0)\,ds_F \\
& -\ \text{Prob}(\theta = 1)\,\text{Prob}(s_F \geq T|\theta = 1) \\
& \left[\left(1 - \frac{\delta}{K}\right)(\mathbb{E}[\theta|ND] + \gamma\,\text{Prob}(I|ND)) + \frac{\delta}{K}(1 + \gamma\,\text{Prob}(I|ND, \theta = 1)) \right] \\
& +\ \text{Prob}(\theta = 1) \int_{s_F \geq T} \left[\left(1 - \frac{\delta}{K}\right)(\mathbb{E}[\theta|s_F] + \gamma) + \frac{\delta}{K}(1 + \gamma) \right] f(s_F|1)\,ds_F.
\end{aligned}
$$

We combine the second term with the fourth term and obtain

$$
\begin{aligned}
\frac{\partial \mathbb{E}[P|\bar{g}]}{\partial \bar{g}} &= -\operatorname{Prob}(s_F \geq T)\left(1 - \frac{\delta}{K}\right)\left(\mathbb{E}[\theta|ND] + \gamma \operatorname{Prob}(I|ND)\right) - \frac{\delta}{K}\operatorname{Prob}(s_F \geq T, \theta = 1) \\
&\quad - \frac{\delta}{K}\gamma\Big[\operatorname{Prob}(s_F \geq T, \theta = 0)\operatorname{Prob}(I|ND, \theta = 0) + \operatorname{Prob}(s_F \geq T, \theta = 1)\operatorname{Prob}(I|ND, \theta = 1)\Big] \\
&\quad + \operatorname{Prob}(s_F \geq T)\left(1 - \frac{\delta}{K}\right)\left(\mathbb{E}[\theta|s_F \geq T] + \gamma\right) + \frac{\delta}{K}\operatorname{Prob}(s_F \geq T, \theta = 1) + \frac{\delta}{K}\gamma\operatorname{Prob}(s_F \geq T) \\
&= \operatorname{Prob}(s_F \geq T)\left(1 - \frac{\delta}{K}\right)\left(\mathbb{E}[\theta|s_F \geq T] - \mathbb{E}[\theta|ND] + \gamma \operatorname{Prob}(NI|ND)\right) \\
&\quad + \frac{\delta}{K}\gamma\Big[\operatorname{Prob}(s_F \geq T) - \operatorname{Prob}(s_F \geq T, \theta = 0)\operatorname{Prob}(I|ND, \theta = 0) \\
&\qquad\quad - \operatorname{Prob}(s_F \geq T, \theta = 1)\operatorname{Prob}(I|ND, \theta = 1)\Big].
\end{aligned}
$$

The first order condition is thus

$$
\begin{aligned}
&\operatorname{Prob}(s_F \geq T)\left(1 - \frac{\delta}{K}\right)\left(\mathbb{E}[\theta|s_F \geq T] - \mathbb{E}[\theta|ND] + \gamma \operatorname{Prob}(NI|ND)\right) \\
&+ \frac{\delta}{K}\gamma\Big[\operatorname{Prob}(s_F \geq T) - \operatorname{Prob}(s_F \geq T, \theta = 0)\operatorname{Prob}(I|ND, \theta = 0) \\
&\qquad - \operatorname{Prob}(s_F \geq T, \theta = 1)\operatorname{Prob}(I|ND, \theta = 1)\Big] - c_D \operatorname{Prob}(s_F \geq T) - c_F \bar{g} = 0.
\end{aligned}
$$

and the second order condition is always negative, so the first order condition defines the true maximum.

Proposition 4.1 shows that the equilibrium value of a threshold T solves

$$
\mathbb{E}(\theta|\mathcal{F}_{public} = ND) + \frac{c_D}{1 - \frac{\delta}{K}} - \xi - \mathbb{E}(\theta|\mathcal{F}_{public} = T) = 0
$$

and the first order condition with respect to $\bar{g}$ at $g = \bar{g}$ that we just derived defines the equilibrium

level of information production effort g:

$$\text{Prob}(s_F \geq T)\left(1 - \frac{\delta}{K}\right)\left(\mathbb{E}[\theta|s_F \geq T] - \mathbb{E}[\theta|ND] - \gamma\,\text{Prob}(I|ND)\right)$$

$$+\ \frac{\delta}{K}\gamma\Bigg[\text{Prob}(s_F \geq T) - \text{Prob}(s_F \geq T, \theta = 0)\,\text{Prob}(I|ND, \theta = 0)$$

$$-\ \text{Prob}(s_F \geq T, \theta = 1)\,\text{Prob}(I|ND, \theta = 1)\Bigg] - c_D\,\text{Prob}(s_F \geq T) - c_F g\ =\ 0\,.$$

In the end, the equilibrium value of T and g solve a system of equations:

$$\begin{cases} F_1(g, T) &=\ 0\,, \\[2mm] F_2(g, T) &=\ 0\,, \end{cases}$$

where

$$F_1(g, T)\ =\ -\left(1 - \frac{\delta}{K}\right)\left(\mathbb{E}(\theta|T) - \mathbb{E}(\theta|ND) + \xi\right) + c_D\,,$$

$$F_2(g, T)\ =\ \text{Prob}(s_F \geq T)\left(1 - \frac{\delta}{K}\right)\left(\mathbb{E}[\theta|s_F \geq T] - \mathbb{E}[\theta|ND] + \gamma\,\text{Prob}(NI|ND)\right)$$

$$+\ \frac{\delta}{K}\gamma\mu - c_D\,\text{Prob}(s_F \geq T) - c_F g\ =\ 0\,,$$

where

$$\mu\ =\ \text{Prob}(s_F \geq T) - \text{Prob}(s_F \geq T, \theta = 0)\,\text{Prob}(I|ND, \theta = 0) - \text{Prob}(s_F \geq T, \theta = 1)\,\text{Prob}(I|ND, \theta = 1)\,.$$

Now let us use the implicit derivation theorem. We have that the derivatives of T and g with respect to some model parameter m are given by

$$\frac{\partial g}{\partial m}\ =\ \frac{\frac{\partial F_1}{\partial T}\frac{\partial F_2}{\partial m} - \frac{\partial F_1}{\partial m}\frac{\partial F_2}{\partial T}}{\frac{\partial F_1}{\partial g}\frac{\partial F_2}{\partial T} - \frac{\partial F_1}{\partial T}\frac{\partial F_2}{\partial g}}\,,$$

$$\frac{\partial T}{\partial m}\ =\ \frac{\frac{\partial F_1}{\partial m}\frac{\partial F_2}{\partial g} - \frac{\partial F_1}{\partial g}\frac{\partial F_2}{\partial m}}{\frac{\partial F_1}{\partial g}\frac{\partial F_2}{\partial T} - \frac{\partial F_1}{\partial T}\frac{\partial F_2}{\partial g}}\,.$$

We will need to calculate

$$\frac{\partial F_1}{\partial c_D} = 1\,,$$

$$\frac{\partial F_1}{\partial c_F} = 0\,,$$

$$\frac{\partial F_1}{\partial \frac{\delta}{K}} = \mathbb{E}(\theta|T) - \mathbb{E}(\theta|ND) + \xi\,,$$

$$\frac{\partial F_1}{\partial T} = -\left(1 - \frac{\delta}{K}\right)\frac{\partial}{\partial T}\Big[\mathbb{E}(\theta|T) - \mathbb{E}(\theta|ND) + \xi\Big]\,,$$

$$\frac{\partial F_1}{\partial g} = \left(1 - \frac{\delta}{K}\right)\frac{\partial}{\partial T}\Big[\mathbb{E}(\theta|ND) - \xi\Big]\,,$$

and

$$\frac{\partial F_2}{\partial c_D} = -\operatorname{Prob}(s_F \geq T)\,,$$

$$\frac{\partial F_2}{\partial c_F} = -g\,,$$

$$\frac{\partial F_2}{\partial \frac{\delta}{K}} = -\operatorname{Prob}(s_F \geq T)\Big(\mathbb{E}[\theta|s_F \geq T] - \mathbb{E}[\theta|ND] + \gamma\operatorname{Prob}(NI|ND)\Big) + \gamma\mu\,,$$

$$\frac{\partial F_2}{\partial T} = \left(1 - \frac{\delta}{K}\right)\frac{\partial}{\partial T}\left[\operatorname{Prob}(s_F \geq T)\Big(\mathbb{E}[\theta|s_F \geq T] - \mathbb{E}[\theta|ND] + \gamma\operatorname{Prob}(NI|ND)\Big)\right]$$
$$+ \frac{\delta}{K}\gamma\frac{\partial\mu}{\partial T} - c_D\frac{\partial}{\partial T}\operatorname{Prob}(s_F \geq T)\,,$$

$$\frac{\partial F_2}{\partial g} = -\left(1 - \frac{\delta}{K}\right)\operatorname{Prob}(s_F \geq T)\frac{\partial}{\partial g}\big[\mathbb{E}[\theta|ND] - \gamma\operatorname{Prob}(NI|ND)\big] + \frac{\delta}{K}\gamma\frac{\partial\mu}{\partial g} - c_F\,.$$

Now for sufficiently small γ we can neglect the terms proportional to γ so that we have

$$\frac{\partial F_1}{\partial c_D} = 1\,,$$

$$\frac{\partial F_1}{\partial c_F} = 0\,,$$

$$\frac{\partial F_1}{\partial \frac{\delta}{K}} = \mathbb{E}(\theta|T) - \mathbb{E}(\theta|ND)\,,$$

$$\frac{\partial F_1}{\partial T} = -\left(1 - \frac{\delta}{K}\right)\frac{\partial}{\partial T}\Big[\mathbb{E}(\theta|T) - \mathbb{E}(\theta|ND)\Big]\,,$$

$$\frac{\partial F_1}{\partial g} = \left(1 - \frac{\delta}{K}\right)\frac{\partial}{\partial g}\mathbb{E}(\theta|ND)\,,$$

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and

$$\frac{\partial F_2}{\partial c_D} = -\operatorname{Prob}(s_F \geq T),$$

$$\frac{\partial F_2}{\partial c_F} = -g,$$

$$\frac{\partial F_2}{\partial \frac{\delta}{K}} = -\operatorname{Prob}(s_F \geq T)\Big(\mathbb{E}[\theta|s_F \geq T] - \mathbb{E}[\theta|ND]\Big),$$

$$\frac{\partial F_2}{\partial T} = \left(1 - \frac{\delta}{K}\right)\frac{\partial}{\partial T}\left[\operatorname{Prob}(s_F \geq T)\Big(\mathbb{E}[\theta|s_F \geq T] - \mathbb{E}[\theta|ND]\Big)\right] - c_D \frac{\partial}{\partial T}\operatorname{Prob}(s_F \geq T),$$

$$\frac{\partial F_2}{\partial g} = -\left(1 - \frac{\delta}{K}\right)\operatorname{Prob}(s_F \geq T)\frac{\partial}{\partial g}\mathbb{E}[\theta|ND] - c_F.$$

Let us now express $\frac{\partial T}{\partial c_D}$, $\frac{\partial g}{\partial c_F}$ in terms of the derivatives that we just calculated for small γ:

$$\frac{\partial g}{\partial c_F} = \frac{\frac{\partial F_1}{\partial T}\frac{\partial F_2}{\partial c_F} - \frac{\partial F_1}{\partial c_F}\frac{\partial F_2}{\partial T}}{\frac{\partial F_1}{\partial g}\frac{\partial F_2}{\partial T} - \frac{\partial F_1}{\partial T}\frac{\partial F_2}{\partial g}}$$

$$= \frac{g\left(1 - \frac{\delta}{K}\right)\frac{\partial}{\partial T}\left[\mathbb{E}(\theta|T) - \mathbb{E}(\theta|ND)\right]}{\frac{\partial F_1}{\partial g}\frac{\partial F_2}{\partial T} - \frac{\partial F_1}{\partial T}\frac{\partial F_2}{\partial g}},$$

$$\frac{\partial T}{\partial c_D} = \frac{\frac{\partial F_1}{\partial c_D}\frac{\partial F_2}{\partial g} - \frac{\partial F_1}{\partial g}\frac{\partial F_2}{\partial c_D}}{\frac{\partial F_1}{\partial g}\frac{\partial F_2}{\partial T} - \frac{\partial F_1}{\partial T}\frac{\partial F_2}{\partial g}}$$

$$= \frac{-c_F}{\frac{\partial F_1}{\partial g}\frac{\partial F_2}{\partial T} - \frac{\partial F_1}{\partial T}\frac{\partial F_2}{\partial g}}.$$

By assumption, we have that the first derivative should be negative and the second one positive.

It follows that

$$\frac{\partial}{\partial T}\left[\mathbb{E}(\theta|T) - \mathbb{E}(\theta|ND)\right] \geq 0,$$

$$\frac{\partial F_1}{\partial g}\frac{\partial F_2}{\partial T} - \frac{\partial F_1}{\partial T}\frac{\partial F_2}{\partial g} \leq 0.$$

(4)

Now let us compute the derivative that we are interested in i.e. $\frac{\partial T}{\partial \frac{\delta}{K}}$ and $\frac{\partial g}{\partial \frac{\delta}{K}}$:

- First,

$$\frac{\partial T}{\partial \frac{\delta}{K}} = \frac{\frac{\partial F_1}{\partial \frac{\delta}{K}}\frac{\partial F_2}{\partial g} - \frac{\partial F_1}{\partial g}\frac{\partial F_2}{\partial \frac{\delta}{K}}}{\frac{\partial F_1}{\partial g}\frac{\partial F_2}{\partial T} - \frac{\partial F_1}{\partial T}\frac{\partial F_2}{\partial g}}$$

$$= \frac{\left(1 - \frac{\delta}{K}\right)\mathrm{Prob}(s_F \geq T)\left(\mathbb{E}(\theta|s_F \geq T) - \mathbb{E}(\theta|T)\right)\frac{\partial}{\partial g}\mathbb{E}(\theta|ND) - c_F\left[\mathbb{E}(\theta|s_F \geq T) - \mathbb{E}(\theta|ND)\right]}{\frac{\partial F_1}{\partial g}\frac{\partial F_2}{\partial T} - \frac{\partial F_1}{\partial T}\frac{\partial F_2}{\partial g}}.$$

We can see that the numerator is negative because $\frac{\partial}{\partial g}\mathbb{E}(\theta|ND)$ is negative, while the denominator is negative because we proved it above. In the end, the derivative is positive,

$$\frac{\partial T}{\partial \frac{\delta}{K}} \geq 0.$$

- Now we compute $\frac{\partial g}{\partial \frac{\delta}{K}}$. To make it easier, let us introduce notations:

$$\Delta_1 = \mathbb{E}(\theta|T) - \mathbb{E}(\theta|ND),$$

$$\Delta_2 = \mathbb{E}(\theta|s_F \geq T) - \mathbb{E}(\theta|ND) = \Delta_1 + \mathbb{E}(\theta|s_F \geq T) - \mathbb{E}(\theta|T).$$

Let us compute the derivative in terms of these notations:

$$\frac{\partial g}{\partial \frac{\delta}{K}} = \frac{\frac{\partial F_2}{\partial \frac{\delta}{K}}\frac{\partial F_1}{\partial T} - \frac{\partial F_2}{\partial T}\frac{\partial F_1}{\partial \frac{\delta}{K}}}{\frac{\partial F_1}{\partial g}\frac{\partial F_2}{\partial T} - \frac{\partial F_1}{\partial T}\frac{\partial F_2}{\partial g}}.$$

The numerator becomes

$$\frac{\partial F_2}{\partial \frac{\delta}{K}}\frac{\partial F_1}{\partial T} - \frac{\partial F_2}{\partial T}\frac{\partial F_1}{\partial \frac{\delta}{K}}$$

$$= \left(1 - \frac{\delta}{K}\right)\mathrm{Prob}(s_F \geq T)\left(-\Delta_1\frac{\partial(\Delta_2 - \Delta_1)}{\partial T} + (\mathbb{E}(\theta|s_F \geq T) - \mathbb{E}(\theta|T))\frac{\partial \Delta_1}{\partial T}\right)$$

$$- \frac{\partial \mathrm{Prob}(s_F \geq T)}{\partial T}\Delta_1(\Delta_2 - c_D).$$

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In the numerator,

$$\frac{\partial \left(\Delta_2 - \Delta_1\right)}{\partial T} = \frac{\partial \left(\mathbb{E}[\theta|s_F \geq T] - \mathbb{E}[\theta|T]\right)}{\partial T} \leq 0,$$

$$\frac{\partial \Delta_1}{\partial T} \leq 0,$$

$$\frac{\partial \operatorname{Prob}(s_F \geq T)}{\partial T} \leq 0,$$

where the first inequality is an assumption in the proposition, the second one was derived in (4) and the last one is an obvious property. In the end, the numerator is positive and thus the derivative is negative. This finishes the proof.

∎